The Effect of Numbers

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The opinions in this paper are those of the author and do not necessarily represent the views or opinions of CORDA or BAE Systems

1 Introduction

1.1 This paper draws lessons from a number of studies on the relationship between numbers and total effectiveness.

2 The Square Law

2.1 Most of you are familiar with the so-called "Lanchester Square Law". It was actually invented in 1902 by Lieut. (later R. Adm.) JV Chase USN, who was a member of the staff of the US Naval War College. He was trying to find a mathematical way of describing ideas about the value of superior numbers in battles between pre-dreadnought battleships. (Reference 1)

2.2 Other related work was published in 1905 by Cdr. (later R. Adm.) BA Fiske USN, and in 1910 by Lieut. Baudry (French Navy). Fiske created a time-stepping model, which produces the same results as the square law if the amount of damage done per time-step is small. Baudry used a time-stepping model to demonstrate the square law. (References 2 and 3) A brief history of the invention of the square law is given in Annex A.

2.3 In the square law assumes:

- That the aim of each side is to shoot at the other, producing many versus many combat.
- That damage occurs gradually – i.e. that the combat modelled is not heavily pulsed.
- That it is possible to distinguish between live and dead enemy.

2.4 Chase, Fiske and Baudry were aware that this was not always the case.

2.5 In Viewfoil 1 there is a fixed force of 1000 units of Red, and the size of the Blue force is varied. The results are after a fixed period of time. As Blue numbers are increased, Blue casualties go down (the blue line), whilst Red casualties go up (the red line). The lines are slightly curved, and become more curved as the duration of the battle increases.

2.6 Viewfoil 2 shows Blue casualties if the battle is fought until Red is exterminated.

2.7 Blue also gains from increased numbers, because the time required to kill a certain proportion of Red decreases. This is shown as the green line in Viewfoil 3. Note that if Blue and Red are equal, it takes infinite time for Red to be exterminated.

3 Battalion Level Engagements

3.1 Let us now look at the results of analyses of real battalion level engagements in past wars. The following graphs are based on relationships developed by David Rowland at DOAE West Byfleet from analysis of infantry and armoured battles (Reference 4). We did some work for PJHQ on this two and a half years ago. I believe more work has been done since then.

3.2 Viewfoil 4 is for an attack by three rifle companies against varying number of good quality defending rifle companies. There is no surprise, the attackers have no prior experience, artillery or mortars do not suppress the defenders, and the fighting is in normal rural terrain. I chose this combination of circumstances because it is a useful starting point to show a number of effects. I think the graphs shows why it is tactically inept for light infantry to attack under these conditions.

3.3 The graph is curved. From the defenders point of view, the larger his force, the smaller the additional benefits of deploying an additional rifle company ('diminishing returns').

3.4 If we look at the lower part of the graph, we can see that the attacker still suffers heavy casualties even when the defender a platoon or less. Now it may be that we are extrapolating the graph too far; but there have been cases from the First World War when two machineguns that somehow were missed out from the bombardment area held up an entire battalion.

3.5 Viewfoil 5 looks at this in another way. The Y-axis is still attacker infantry casualties, but this time expressed as absolute numbers, and not as a percentage. The X-axis is the number of attacker rifle companies. If we look at the green line, which is for attacking a single rifle company, we can see that by using a smaller force the attacker can reduce his casualties; admittedly he will suffer a higher percentage casualties; but his absolute casualties will be lower. This is known as the 'principle of economy of force'. It is found in real life, but not in the square law nor in time-stepping simulations that work like the 'square law'.

3.6 The blue line shows attacker casualties when the defence consists of a rifle platoon. The red line is for an attack on a rifle section.[1]

3.7 Let's go back to our previous graph (Viewfoil 4). If we now plot attacker casualties for a similar attack by three armoured infantry companies, it produces the blue line on Viewfoil 6. It is 'S' shaped. If we are attacking at three to one, the fire from the MICVs will probably suppress the defending machineguns; and our expected casualties are virtually zero. Only if the defenders have more than one than one rifle company will attacker casualties rise; we then start to get the same 'diminishing returns' curve as with the light infantry. Notice how the graphs get closer together as the number of defenders rises. For the attacker, the value of being better equipped is high in when he outnumbers the defence; but declines as defender numbers become more overwhelming.

3.8 In the next graph (Viewfoil 7) we look at an attack by three attacker light infantry companies and 36 MBTs against one defender rifle company and varying numbers of defender ATGWs. The attacker MBTs suppress defender machineguns, reducing attacker casualties from 37% (in the previous graph), to 5% when there are no defender ATGWs. The defender ATGWs protect the infantry from the MBTs, reducing this effect. So the more ATGWs there are, the more the attacking infantry suffers from defending machineguns. If there were a very large number of ATGWs, attacking infantry casualties would rise to the 37% it was when there were no MBTs.

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1 Each rifle section has been assumed to possess two light machine guns. This incidentally shows one of the limitations of analytical modelling. It would be difficult to set up an attack by three rifle companies on a section, where the section got to engage all of them. It is more likely that only part of the attacking force would be engaged; perhaps a platoon or possibly a company.

3.9 Viewfoil 8 shows both attacker MBT and infantry casualties. Notice the change in both the X and Y scales. From the point of view of increasing attacker infantry casualties there is very little benefit in having 50 instead of 25 defender ATGWs. But from the point of view of knocking out more tanks there is. This illustrates how choosing a different measure of effectiveness can affect your perception of how many you need.

3.10 There is incidentally another synergy between the ATGWs and infantry. If there were more defender infantry companies, the ATGWs would kill more tanks; you would therefore need less ATGWs to kill the same number of tanks.

3.11 Viewfoil 9 shows the effect of inferior anti-tank weapons. They don't kill as many tanks for same number of weapons.

4 Helicopter Mission Modelling

4.1 The next set of graphs is from some modelling of helicopter operations that we undertook for CDA/Land-Air in the last twelve months. The model is called the Helicopter Operations Model, and was written by my colleague Michael Young.

4.2 In the study we ran this model for a large number of COEIA options over about 20 vignettes. The vignettes were set in different parts of the world, with different climates, and contained all the missions that we would like the helicopters to undertake over a period of one to three days.

4.3 Viewfoil 10 shows the effectiveness of different numbers of helicopters in one of these vignettes, for two options. You can see that the 'Brown Option' is better in this vignette than the 'Red Option' (it carries a higher payload). Neither of them can complete all the missions, no matter how many you deploy. That does not matter because they can achieve the desired level of effectiveness anyway.

4.4 This graph is rather like the one we have just seen for ATGWs. It shows 'diminishing returns'. Both options have the same maximum level of effectiveness; with the better option you need less helicopters to get there.

4.5 The reason the graphs are the same shape is that:

- With the ATGWs there are a limited number of tanks to be knocked out, and a limited number of oportunities to do so.
- With the helicopters there are a limited number of missions to be undertaken, and each mission has a limited window in which it must be undertaken.

4.6 Viewfoil 11 shows two more options: 'Blue' and 'Green'. They do not have the lift in these climatic conditions to carry the fuel and payload to complete missions that 'Brown' and 'Red' can. There maximum level of effectiveness is lower, and it takes a lot of them to reach it. For this vignette, neither 'Blue' nor 'Green' can reach the desired level of effectiveness.

4.7 There are certain items of special equipment that are necessary to undertake certain missions. Viewfoil 12 shows the effect of differing numbers of one such special equipment.

Red Shows effectiveness if none of the helicopters have this special equipment.

Blue Shows effectiveness if one of the helicopters has this special equipment.

Green Shows effectiveness if two of the helicopters have this special equipment.

Brown Shows effectiveness if four of the helicopters have this special equipment.

Purple Shows effectiveness if all of the helicopters have this special equipment.

4.8 Notice that, in this vignette, if you have 30 helicopters, there is no advantage in having all the helicopters equipped with this item of special equipment over having only four so equipped. This is because with 30 helicopters in total (including four with the special equipment), there will always be enough not committed to other missions to undertake missions, to always be able to undertake the missions that need the special equipment. With less than 30 helicopters in total (but including four with the special equipment), some of the specially equipped helicopters are busy on other missions – in that case there is a slight advantage in having more than four so-equipped.

4.9 Many items of special equipment are heavy, they sometimes add drag, and in one case reduce the engine-power. For most vignettes, it is better to have a mixed fleet with some equipped with the special equipment, and some not equipped. Those without the special equipment have a better load-carrying ability; those with can undertake missions that the others cannot.

5 Strategic Bombing

5.1 How Many Strategic Bombers Were Required in 1939

5.1.1 In the First World War, the Germans undertook what by later standards was a minimal level of strategic bombing against England. It caused considerable disruption, mainly because of the air raid precautions that were considered necessary. We also had to deploy fighters and build anti-aircraft guns; in other words it diverted resources that could have been used on the Western Front.

5.1.2 If you believe that it is possible to win a war through strategic bombing, there must be a level of bombing effort, which will win, and anything less than that will only cause disruption and divert resources. Obviously the level of effort required, depends on how it is used, and also on the consequences for the enemy of giving in.

5.1.3 In the 1930s some politicians believed that it was possible to win wars by air power alone, through strategic bombing (Reference 5). In 1937 RAF Bomber Command was invited to look at a number of plans for how they would do this. One was the plans was 'WA.5', which was the attack of the German War Industry including the supply of oil with priority to that in the Ruhr, Rhineland and Saar.

5.1.4 Precision Bombing. Bomber Command thought that it not likely to be effective if undertaking some of the other plans. But their appreciation was that 'WA.5' could work if they put out of action 26 coking plants and nineteen power stations listed in an intelligence report to Bomber Command. They estimated that this would take 3,000 sorties in a fortnight for the predicted loss of 176 aircraft. If it could be achieved, they thought it would bring the German war-making power almost to a standstill. The Air Ministry thought that this was too optimistic because of the difficulty of hitting power plants and destroying coking plants.[2] (References 6 and 7)

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[2] After the war it emerged that in choosing to target electrical power, Bomber Command had selected a highly vulnerable part of the German industrial machine. "It may be doubted, however, whether Bomber Command was ever sufficiently accurate in its bombing to destroy so many targets of such small dimensions though experience showed that

5.1.5 In September 1939, Bomber Command owned about 280 medium bombers, of which about 200 were available.[3] They estimated that they needed 3,000 sorties in a fortnight. Using data from World War II on sortie rates and availability for kinds bombers involved, I estimate that they would have needed to own between 1,200 and 1,500 medium bombers to do this.

5.1.6 However, this figure of 3,000 sorties assumed a mean error of 300 yds. At AORG West Byfleet for studies in the 1950s they assumed a mean error of 800 yds. This increases the number of sorties required to 21,333. The German air defences would have caused some bombers to abort their missions without bombing. If we take this into account, the number of sorties required rises to 28,000.

5.1.7 The loss rate assumed by Bomber Command was 5.87%. That is not too bad. The 8th US Army Air Force (8 USAAF) B-17s that bombed Hamburg in daylight raids in 1943 took 6.94%. However 5.87% of 28,000 sorties produces a loss of 1,650 aircraft, whilst 6.94% means a loss of 1,950.

5.1.8 Based on these revised figures, the number of medium bombers that Bomber Command would have needed to win the war using plan WA.5 rises to between 11,250 and 16,100 medium bombers. It is difficult to see the British taxpayer providing these.

5.1.9 Area bombing of cities. Let us look at what would have been necessary to win the war by 'terror bombing' of cities. This was the method of strategic bombing that British politicians were afraid of in the 1930s (Reference 5). The RAF ended up doing it, because they could penetrate German air-space by night, but not by day for most of World War II.

5.1.10 The bombing of Hamburg from 24 July to 3 August1943 was one of the more successful operations by Bomber Command (with help from 8 US Army Air). This city showed up exceptionally well on the terrain mapping radar carried by some of our bombers. The bombing was therefore unusually concentrated, and at least 40,000 people were killed. This raid terrified the German government. "We were of the opinion", said Albert Speer after the war, "that a rapid repetition of this type of attack on another six German towns would inevitably cripple the will to sustain armament manufacture and war production. It was I who reported to the Fuhrer at that time that a continuation of these attacks might bring a rapid end to the war." (Reference 12 p176)

5.1.11 One can calculate the number of the bombers required to do this in 1939 and the spring of 1940. It would have been possible to get the concentration of bombing needed if the RAF had bought the kinds of radio beam based navigational equipment used by the Germans at that time. Every bomber should have been fitted with it. Of course you needed France to put the transmitting stations in.

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considerable damage could be done by relatively small attacks." "But it was hardly thought by the Air staff that such great feats could be accomplished until bombers were obtained which by their speed and armament would be able to penetrate to the Ruhr in daylight and to be able to carry out precision bombing. Meanwhile, therefore, the force must be conserved for the future rather than risk its immediate destruction. Otherwise when the heavy bombers appears, there would be no crews ready to use them." "On 21st June 1938 the Prime Minister announced in the House of Commons that Britain would only bomb purely military objectives and even so would take due care to avoid civilian casualties."

To solve the problem of penetrating German air-space, Sir Edgar Ludlow-Hewitt (CinC Bomber Command) advocated developing long range fighters to escort bombs and adding to the defensive armament of bombers.

[3] The figure of 280 comes from Reference 8 page 20. At 1800 hrs on 9 September 1939 Bomber Command had an 'operational strength' of 67 Wellingtons, 60 Whitleys and 75 Hampdens, a total of 202 (Reference 9).

5.1.12 We did not develop the navigational equipment until after we changed our doctrine from day to night bombing. We would also have needed to train their crews in night flying.[4]

5.1.13 Let us assume that Bomber Command would have attacked other targets, as it did in the 'Battle of Hamburg'. It could have inflicted about one Hamburg a month with a force of 5,000 medium bombers. Bomber Command would have needed to own about 8,000 bombers, and would have lost about 3,000 of them. There would have been about 300,000 German casualties.

5.1.14 Conclusion. Because Bomber Command never had enough bombers to win the war through strategic bombing, they were stuck at the disruption and diversion of resources level of effectiveness. It is perhaps worth mentioning that up to the end of 1941, the ultimate objective of the Air Staff was a frontline strength of 4,000 heavy bombers (Reference 12 p92). Given the right doctrine and a solution to the navigational problems, that might have been enough to win the war. The problem was that with the available technology they needed France to put the ground stations for navigational equipment.

5.2 The Effect of Concentration on Civilian Casualties

5.2.1 When Bomber Command bombed German cities in World War II, they used a mixture of high explosive (HE) and incendiaries. People on the ground put out fires. If you could put enough bombs in a small area, you could overwhelm these people. With heavy enough bombing, HE would drive these people into shelters, and with a bit of luck knock out water mains. If bombing was scattered, the number of people available to deal with the effects of bombing was larger.

5.2.2 If bombing was concentrated enough, you could get fires building up and joining such that they sucked in air from outside, producing a 'fire-storm' as in Hamburg on 27/28 July 1943.

5.2.3 This kind of bombing is therefore an example of improving returns. If you can drop a large number of bombs in a small area, you get a much better pay-off in terms of civilian casualties per bomb.

5.3 How the German Night Fighter Defences Worked in 1942

5.3.1 In the 1930s, both Britain and Germany put resources into creating a system that would defeat day bombing. Because they were fairly successful at that, the RAF and the Luftwaffe were forced to turn to night bombing.

5.3.2 In 1941, the Germans created a method that is now called Ground Controlled Interception (GCI). They had a series of GCI stations at 20 mile intervals, that RAF bombers had to cross. Each station had three radars: a search radar, and two tracking radars, one for the bomber to be intercepted and one for the fighting that was guided on to the bomber by the GCI station.

5.3.3 The British also developed GCI. However, British tracking radars were made less accurately than their German equivalents. This meant that using separate radars to track the fighter and the bomber would not have worked with British radars. The British were therefore forced to use the same radar for both tasks. Given that what you wanted to know was the relative position of the fighter and bomber, errors could be tolerated as long as the same error applied to both. In other words, because the British had worse tracking radars than the Germans, they needed half as many to do the same job to the same standard.

5.3.4 The German ground controlled interception stations had three weaknesses:

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4 At the start of World War II, about a third of the RAF's medium bomber crews were trained in night flying.

- Each station could only control one fighter intercepting one bomber at once. This meant that they could be swamped, by having a great mass of bombers flying together.

- It also meant that only a limited number of fighters could operate at once given the way Bomber Command operated. At the GCI system's zenith in June 1943, the limit was about 60 fighters. (Reference 13 paragraph 16)

- Finally there was the possibility that the radars would be jammed. Effective jamming started on 25 July 1943.

5.4 Swamping of German Air Defences in 1942

5.4.1 In 1942, the RAF tried some 'thousand bomber raids' against Germany. These raids required a maximum effort and even had aircraft from Operational Training Units (OTUs) and Coastal Command. Viewfoil 18 shows bomber losses on the 'thousand bomber raids' and other large raids involving OTUs against Germany between May and September 1942. There seems to be a pattern.

5.4.2 When you plot all the night raids that Bomber Command's main force undertook against Germany between April and October 1942 a different pattern emerges (Viewfoil 19). The raids in which the OTUs participated are marked in red. The simplest relationship, that bomber losses are proportional to the number of sorties despatched, provides a good fit.

5.4.3 Swamping of defences is going on. A study by Bomber Command's Operational Research Section showed that losses of aircraft that carried better navigational equipment (Gee) were 40% less than those of aircraft without this equipment. This was because bombers with better navigational equipment were better at keeping in the mass of bombers and therefore benefited from the ffect of swamping. (Reference 13 paragraph 10)

5.4.4 The problem is that there were quite a lot of effects going on, and many of these effects cancelled each other out. It shows nicely the danger of looking at small samples of data that show precisely the effect you would expect.

5.4.5 It is probably worth mentioning that there are no square law type effects here. One night fighter attacked one bomber at a time. When a bomber saw a fighter, the preferred course of action was to evade. There was no concentrating of fire from bomber formations on fighters like there was in day bombing.

5.5 How the German Night Fighter Defences Worked Later in the World War II

5.5.1 As mentioned before, effective jamming of the German GCI system started on 24 July 1943. This was done using chaff. The Germans kept the GCI stations in use; they were still effective against stragglers and forced Bomber Command to jam them. The Luftwaffe were still able to track the bomber stream, partly by following the jamming and partly by having commentary aircraft fly above the bomber stream. Night fighters then joined the bomber stream and selected their own targets; Bomber Command called this 'freelancing'.

5.5.2 From the German point of view this had the benefit that much larger numbers of fighters could operate at once. By late 1943, typically 300 fighters would be operating (as compared with 60 under GCI earlier in the year). (Reference 13 paragraph 16) This is another example of a technical difficulty forcing people to make more efficient use of existing resources.

5.5.3 Fortunately for Bomber Command the kill rate per fighter was lower when 'freelancing' than under GCI. ORS Bomber Command reckoned that against the same force of bombers, the 60 GCI fighters shot down approximately the same number of aircraft as the 300 freelance fighters.

Clearly, the Germans could have benefited from a combination of GCI with the 'freelance' method, as that would have kept more of the available night-fighters operating at once.

6 Skill and Limitations on Numerical Superiority

6.1 This section is based Chapter Two of "One War Today", by Gen. F von Bernhardi, which was publish in English in 1912 (Reference 14).

6.2 If a force has superior numbers, it may not be able to gain the full advantage of its numbers due to:

- Lack of space to deploy in,
- Lack of time to deploy all its force,
- Insufficient logistic support,
- Technical problems or
- Poor organisation.

6.3 It is also worth pointing out that the addition of reservist troops or battle casualty replacements to a unit may (at least temporarily) reduce the unit's overall effectiveness because the decline in cohesion may outweigh the increase in numbers.

6.4 Gen. F von Bernhardi advanced the theory that "a general may neutralize the superiority of an enemy, if the proportionate numerical strength of both sides leave any chance at all of inflicting on the enemy, one way or the other, losses large enough to neutralize his superiority. But if the numerical superiority of the one party is so great as to preclude the weaker party from decisively affecting, even by possible successes in the tactical defensive or by successful offensive actions, the total effect of the enemy's numbers, then no generalship avails to neutralize the effect of such superiority. *This is the most essential law of numbers.*" [Italics in original.]

6.5 This concept of there being a range of force ratios over which either side may win is something we have used a study for MoD of the validity of a system of static measures (References 15 and 16). We called this range of force ratios, 'the zone of uncertainty'. Most conventional land campaigns are fought within this range of force ratios (Viewfoil 20). This is not surprising; if the attacker did not think he could win, he would not attack; if the defender did not think he could win, he would withdraw.

6.6 If one takes account of qualitative factors in calculating the force ratio, the zone of uncertainty shrinks. Important qualitative factors include the skill of troops, surprise, and air superiority (Viewfoil 21).

7 Summary

7.1 The relationship between numbers and aggregate effectiveness depends on the measure of effectiveness.

- In the example of ATGWs engaging tanks, we saw that aggregate effectiveness at killing tanks continued to rise sharply as ATGW numbers increased, even at numbers when effectiveness at protecting infantry (thus allowing them to inflict more casualties on attacking infantry) had plateaued.

- In the example of strategic bombing, effectiveness at causing disruption and diverting enemy resources increased with increasing bomber effort, but because there were insufficient bombers to win the war on their own, they failed just as absolutely when Bomber Command had a front-line strength of 250 medium bombers in 1939, as when its front line strength was 1,100 in the middle of 1943.

7.2 In a situation when there are limited windows of opportunity, it is common to see a diminishing returns effect as numbers increase. We saw this with ATGWs engaging tanks and with helicopters supporting the army. With the German night fighters, limits on the number of GCI stations crossed by British bombers created the same effect for increasing numbers of night fighters.

7.3 There are also inefficiencies in increasing force size. This again can create diminishing returns.

7.4 Technical deficiencies sometimes force people to change the way they operate in an effort to compensate; this may lead to their finding ways of using available resources in a way that is (in some ways) more efficient. We saw this with British tracking radars in their GCI stations, and later with the German introduction of 'freelance' night fighters in response to British jamming of GCI stations' tracking radars.

7.5 Increasing the number of targets for the enemy to shoot at, increases enemy effectiveness in some respects. In some situations the targets combine together to counter-act this, in others they do not.

7.6 Finally there are limits on the ability of skill, either of troops or generals, on their ability to counter-act superior enemy numbers.

8 References

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17. These articles are reproduced in "Aircraft in Warfare, the dawn of the fourth arm", by FW Lanchester, pub Constable, 1916.
18. "The Effect of the Quantitative Strength of Fighting Sides on their Losses", by M Osipov, pub Voennii Sbornik, Vol 3-7, 1915, An English translation was issued by the Soviet Army Studies Office at Fort Leavenworth in 1987.

9 Annex A – History of the Square Law

9.1 The square law was invented in 1902 by Lieut. (later R. Adm.) JV Chase USN, as a mathematical way of describing ideas about the value of superior numbers in battles between pre-dreadnought battleships. (Reference 1) For a battle fought to the destruction of one side, Chase's equation was as follows:

$$\frac{TotalDamagetoBlue}{TotalDamageto\,\mathrm{Re}\,d} = \frac{AveLifeBlue}{AveLife\,\mathrm{Re}\,d} \times \frac{NoBlueShips - \sqrt{\left(NoBlueShips^2 - \frac{AveLife\,\mathrm{Re}\,d \times AveDestructivePower\,\mathrm{Re}\,d}{AveLifeBlue \times AveDestructivePowerBlue} \times No\,\mathrm{Re}\,dShips^2\right)}}{No\,\mathrm{Re}\,dShips}$$

9.2 If one assumed that each sides' ships are identical, then this was simplified as follows:

$$\frac{TotalDamagetoBlue}{TotalDamageto\,\mathrm{Re}\,d} = \frac{NoBlueShips - \sqrt{\left(NoBlueShips^2 - No\,\mathrm{Re}\,dShips^2\right)}}{No\,\mathrm{Re}\,dShips}$$

9.3 Chase's equations were recorded in a secret memorandum at the US Naval War College, and it is likely that very few people heard of them at the time. (Reference 1)

9.4 BA Fiske. In 1905, the Journal of the US Naval Institute published an article by Cdr. (later R. Adm.) BA Fiske USN containing the time-stepping model shown in Table 1. In a battle fought to the destruction of one side, if the time-steps are small, Fiske's model produces virtually identical results to Chase's. The Fiske approach had the advantage of being more flexible and easier for naval officers to understand. (Reference 2)

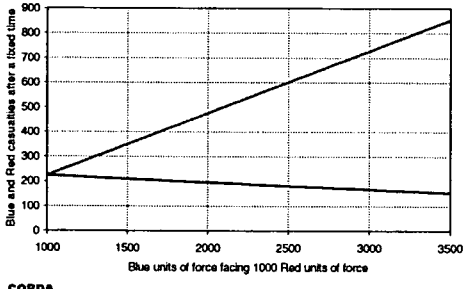
9.5 Figure 1 shows the effect of numbers on Blue casualties in a battle fought to the annihilation of Red, from either of these models.

9.6 A Baudry. In March 1910, a French naval officer Lieut. A Baudry completed a book (Reference 3) used time-stepping models to demonstrate that in a battle between N and N_1 similar battleships, their aggregate fighting power was in the ratio of $N^2 : N_1^2$. This was published in English in the Spring of 1914. (Reference 3)

9.7 FW Lanchester. FW Lanchester published his ideas on the effect of numbers in October 1914 (Reference 17).

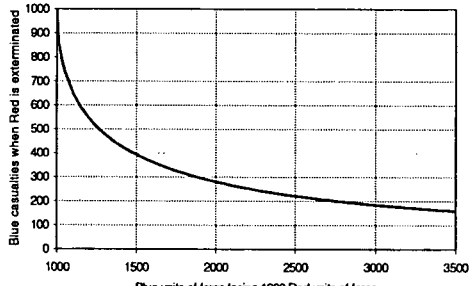
9.8 M Osipov. In 1915 a Russian, M Osipov published a series of articles, containing an analysis of data on casualties in land battles between 1805 and 1905. Osipov also proposed a square law like Chase, and used a Fiske-type time-stepping model to show why this should be so (Reference 18).

Square Law Results after a Fixed Time



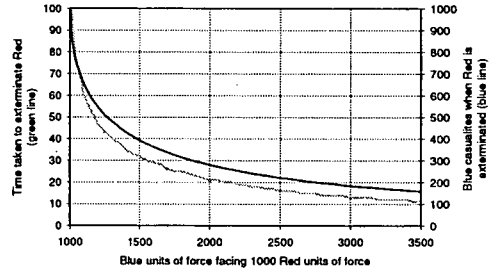
CORDA 1

Square Law Results When Red is Exterminated



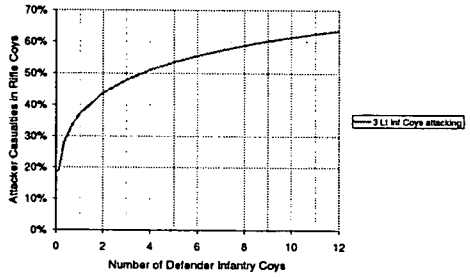
CORDA 2

Square Law – Time Taken to Exterminate Red Compared with Blue Casualties



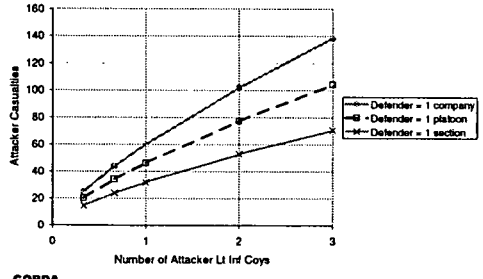
CORDA 3

Model Based on Analysis of Combat Data – Effect of Varying Size of Defender Force



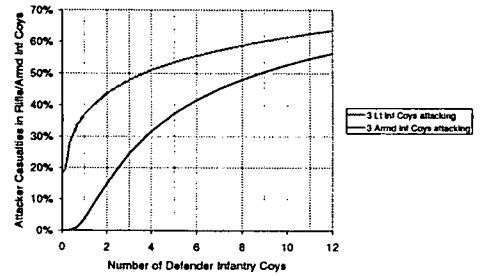
CORDA 4

The Value of Economy of Force



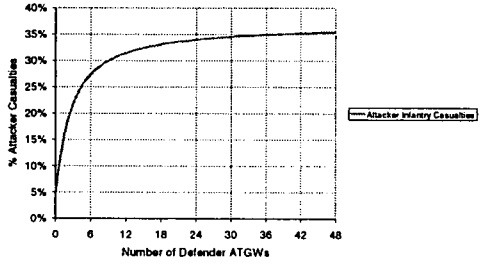
CORDA 5

Model Based on Analysis of Combat Data – Effect of Varying Size of Defender Force



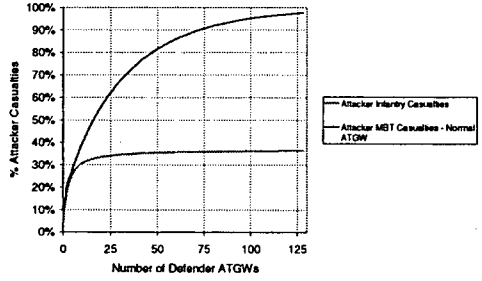
CORDA 6

Effect of Quantity of AT Weapons on Attacker Infantry Casualties



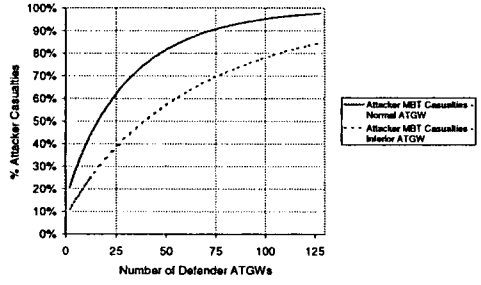
CORDA 7

Effect of Quantity of AT Weapons on Attacker Infantry and Armour Casualties



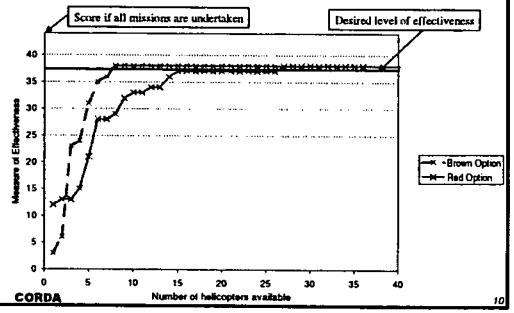
CORDA 8

Effect of *Quality* of AT Weapons on Attacker Armour Casualties



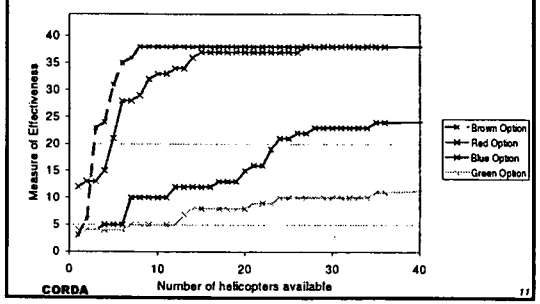
CORDA 9

Effectiveness of Different Helicopter Options



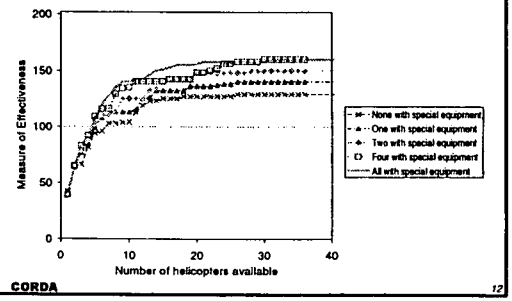
CORDA 10

Effectiveness of Different Helicopter Options – Including Two Options of Low Effectiveness



CORDA 11

How Differing Numbers of Special Equipment Alter Effectiveness



CORDA 12

RAF Bomber Command Plan WA.5

- The attack of the German war industry.
- Knock out 26 coking plants and 19 power stations
- Estimated – 3,000 sorties in two weeks, loss of 176 aircraft.

RAF Bomber Command Plan WA.5

- In September 1939, Bomber Command owned 280 medium bombers.
- Estimated – 3,000 sorties in two weeks, loss of 176 aircraft.
  - Air Staff said this was too optimistic
- Needed to own 1,200 – 1,500 medium bombers.

What Was Really Needed to do RAF Bomber Command Plan WA.5?

- Estimated Sorties in Two Weeks
  - 3,000 sorties
  - 21,333 sorties (takes account of greater inaccuracy)
  - 28,000 sorties (takes account of greater inaccuracy and higher early return rate)
- Estimated Losses
  - 5.87% (est by Bomber Command for day bombing)
    - Loss of 1,650 aircraft
  - 6.94% (8 USAAF in day bombing of Hamburg 1943)
    - Loss of 1,950 aircraft
- Needed to own 11,250 – 16,100 medium bombers.

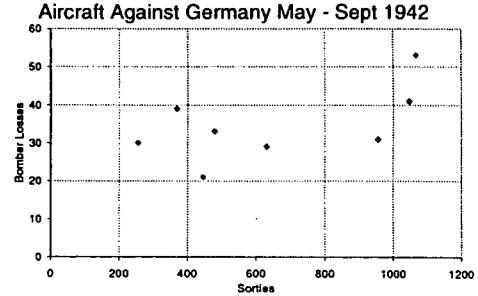
How Many Bombers Would RAF Bomber Command Have Needed to Win the War in 1939/40 by Attacking Cities

- Needed to own 8,000 medium bombers
  - Force size of 5,000 bombers
  - Casualty replacements = 3,000 bombers
- 7 months
- About 300,000 German casualties
- Needed:
  - Doctrine to attack at night (and training)
  - Navigation equipment (and France to put ground stations for navigational equipment)

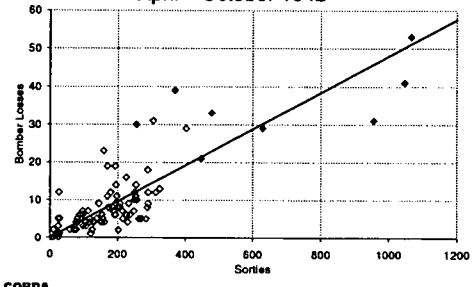
War Winning in 1939/40 with RAF Bomber Command

- Owned 280 medium bombers
- Needed 11,000 – 16,000 to win with precision bombing
- Needed 8,000 to win by attacking cities
  - But this would only work if we had France to put ground stations for navigational equipment

Bomber Command Losses on the 1000 Bomber Raids and Other Large Raids Involving OTU Aircraft Against Germany May - Sept 1942

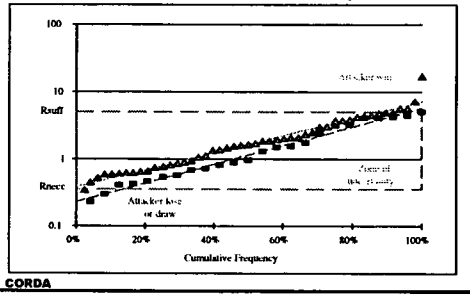


Bomber Command Losses on Main Force Night Bombing Raids Against Germany April – October 1942



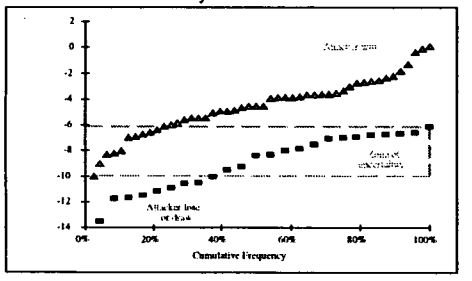
CORDA 19

Most Land Campaigns are Fought at Force Ratios Where Either Side Could Win Given Equal Skill and Generalship



CORDA 20

If the Force Ratio Takes Account of Qualitative Factors, Many Campaigns Were Fought at Ratios Where Only the Attacker Could Win



CORDA 21