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NONLOCALITY, NONLINEARITY AND COMPLEXITY: ON THE MATHEMATICS OF MODELLING NCW AND EBO

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ABSTRACT

The concepts of network centric warfare (NCW) and effects-based operations (EBO) involve the use of complex systems (networked forces or networked multi-agencies), which may consist of many components that are highly heterogeneous in functionality and capability, and that their interactions can often be both nonlocal and nonlinear. In our effort to understand the behaviours of these systems and to analyse the potential benefits of these concepts, it is extremely important to ensure that the mathematics underpinning the modelling and analysis indeed has a requisite level of representation of the interactions and of the underlying symmetries of the systems. A closely related challenge is to develop modelling and simulation tools with the ability to help the analysts understand the origins of all simulation results, may they be genuine emergent properties, artefacts or simply expected outcomes, and answer unambiguously the questions of *why* and *how*.

INTRODUCTION

Network Centric Warfare (NCW) is fundamentally about using cooperative efforts to bring spatially separated warfighting capabilities together and project the desired effect to the right place at the right time. That is, it involves long-range interactions and propagation of nonlocal effects. Similarly, the concept of effects-based operations (EBO) involves utilising capabilities and exercising effects that may be separated across space, time and different domains of the DIME (diplomatic, information, military, and economic) construct. These complex nonlocal interactions can have far-reaching impact on our understanding of the operational outcomes in a number of ways. First, they could lead to nonlinear behaviours and outcomes that would be impossible to understand or even approximate unless the nature of the interaction is properly taken into consideration. Second, these long-range interactions, together with other (*e.g.* short-range) interactions, and the patterns of interaction (*e.g.* the networking structure) amongst the players in a networked force will change with changing mission demands and circumstances. Realistic mathematical models of NCW and EBO must have the requisite form to represent the nonlocal nature of these complex interactions, and the associated simulations must be able to track the time-evolution of the interactions and their changing relational structure.

When one is facing the task of determining the requirements of tools for NCW modelling and analysis, the common criterion is that the tools are able to model some kind of information exchange between the nodes. What is seldom questioned is the mathematics underpinning not only the information exchange modelling but also the functional form of interactions between the nodes. For example, in models based on cellular automata, the mathematics governing the action of each automaton or agent is by definition localised in both space and time. Therefore, attempts to simulate information exchange between agents separated by distances greater than the time-step size (*i.e.* effective interaction distance) will amount to nothing more than modifying the initial conditions of the function governing the next moves of the agents receiving the information. Although the modified inputs may have altered the behaviours of the agents, communications between agents separated by a distance greater than the interaction distance have not in reality been modelled because the localised functional form cannot take the long-range interactions into account.

Like many fields of contemporary scientific research, military operational research has become a highly multidisciplinary endeavour. Ideas and theoretical models developed in diverse disciplines may often be employed in solving seemingly unrelated problems that can be cast into similar conceptual frameworks and mathematical formulations. In this paper, we will first examine examples of physical and biological systems in which certain complex properties cannot be explained in terms of short-range interaction models. We will pay special attention to the subtle but important relationship between the range of interactions and the characteristic symmetries in these systems. As one may expect, the more localised the interactions, the easier it is to represent the system with a higher degree of symmetry and a simpler mathematical formulation. We will then turn our attention to network modelling and show that similar considerations also apply in network analysis in general and NCW modelling in particular, where great care must be exercised to ensure that the mathematics underpinning the modelling is compatible with the problems at hand.

Finally, the author will briefly describe a new modelling and simulation (M&S) tool being developed in the Defence Science and Technology Organisation (DSTO) for modelling and analysing not only the value of information in a mission, but also the impact that different command and control (C2) and networking structures may have on the operational effectiveness of a networked force. This M&S tool, the *Dynamic Agents Representation of Networks of Systems* (DARNOS), has been designed to be able to model interactions at arbitrary distances (in either graph-theoretic or spatial sense), and to track the time-evolution of these interactions and their relationships, in a networked force. This modelling and simulation capability has the pleasing feature of being able to offer complete *visibility* and *traceability*, so that the reasons for unexpected (emergent) effects or interesting results emerging from the complex interactions can be properly examined and understood.

PHYSICAL SYSTEMS

A classic example of long-range interactions leading to nonlinear properties, which cannot be explained in terms of short-range models, is the well-known nonlinear dependence on alloy composition of the spin-orbit coupling in II-VI and III-V semiconductors alloys. This problem has serious implications on the ability of scientists to accurately predict important new alloy properties including the band-gap. The origin of the problem had defied proper understanding for a couple of decades during which physicists were experimenting with different variations of short-range models. The nonlinearity was eventually explained when long-range effects and the commensurate reduction in crystallographic symmetry were explicitly included in the mathematical and computational modelling [*Ling and Miller 1986, 1988*].

In a similar fashion, first-principles theoretical calculations of a class of itinerant magnetic metal alloys successfully explained the origins of some puzzling neutron scattering data on these materials [*Ling et. al. 1995, Staunton et. al. 1997*]. The underlying mechanism driving the nonlocal, low symmetry scattering patterns has been traced to the competing interactions between the magnetic and atomic ordering forces. Although both the atomic and magnetic ordering forces are short-range in nature, their competing interactions have led to the observed (emergent) nonlocal interaction patterns. Indeed, this class of magnetic materials exhibits true emergent behaviour in the complexity-theoretic sense, in that the short-range interactions, each of a higher degree of symmetry, give rise to long-range behaviour with reduced symmetry. Of special interest to this Symposium is the fact that in some of these magnetic systems the atomic ordering is driven by the state of magnetisation via the underlying "electronic glue". The possibility to achieve a goal (*i.e*. a particular atomic ordering state) by different means (*i.e*. the magnetic fields) via the subtle interplay between atomic and magnetic ordering forces can be viewed as one of Nature's examples of effects-based operations.

There are many other physical phenomena in which global interaction patterns arise out of short-range interactions, from interference and diffraction patterns to the formation of turbulence in fluids and the well-known natural soliton phenomenon (tidal bore) in the River Severn. Optical interference and diffraction patterns occur as individual photos of different phases reinforce or cancel each other at the point of pattern formation. Similarly, though the formation of long-range order (turbulence, solitons) is the direct result of short-range interactions (*i.e*. the viscous forces between water molecules), short-range models alone can neither predict nor explain the emergent global pattern of collective interactions. Finally, it is worth noting that, in all the cases considered above, the range of interaction is intimately related to the underlying symmetry in the system under consideration. In the case of the magnetic alloys, the symmetry is broken by the presence of magnetic fields, whereas turbulence and solitary waves emerge with low symmetries from the high symmetry state of interacting water molecules.

BIOLOGICAL SYSTEMS

In the first couple of decades after the structure of DNA was discovered, many scientists attempted to model the melting and folding of DNA protein chains in terms of pair-wise interaction models, such as variations of the Ising model, assuming that the processes were solely driven by interactions between the neighbouring base pairs. It was soon found that in many DNA protein chains the folding and melting processes could involve tens or hundreds, even thousands, of molecules manoeuvring themselves with high precision in 3-dimensional space. This fact has two immediate implications. First, given the number of molecules involved simultaneously in the action of folding or melting, it would be unrealistic to expect short-range models to be able to describe the processes satisfactorily. Second, it is also clear that some kind of long-range interactions are responsible for the highly complex but precise folding actions in 3-D to achieve the stable final topologies [Chan 1998]. Furthermore, it has been argued that the combined actions of both short and long-range interactions are responsible for the wide range of folding speeds in DNA protein chains [Chan 1998; Gromiha *et. al*. 2002; Knott *et. al*. 2004]. As in the case of physical systems, the degree of symmetry and the range of interaction are closely linked in biological systems: Pair-wise interactions between DNA base pairs possess a higher degree of symmetry than that in long-range interaction models, and models that encapsulate competitions between long and short-range interactions are likely to require functional forms of a still lower degree of symmetry.

NETWORK SYSTEMS

In most networks analyses, from the Internet to electric power grids, from biological molecules to social networks [Greene and Higman 2003, Strogatz 2001, Robins *et. al*. 2005], the focus has been on characterising the networks by focusing on the number of nodes each node is connected to, and the average path length between nodes. In this approach, no differentiations are made of the capability and functionality of the nodes or links. Nor is attention, in most cases, being paid to the dynamic nature of many networks in which links are continuously formed, broken and reconnected as time evolves. Despite these assumptions and limitations, one can gain surprisingly a rich insight into the characteristics of the networks, and classify them into general types such as regular, small-world, scale-free or random networks.

These studies have given us much valuable knowledge of their general properties and behaviours. For example, one can gauge a network's robustness against, or susceptibility to, random failures or targeted attacks, based on the afore-mentioned analytical approach.

A rarely noted but profoundly significant difference between military networks and the majority of networks studied in the literature is the fact that military networks are formed for specific reasons. A networked force (*e.g.* an Agile Mission Group) is a team or an organisation with goals (missions) shared by its members, though each member will have their own individual role to play and objective to achieve. This view is consistent with the Mintzberg picture of organisations [Mintzberg 1979]. An analogy may be drawn here between an ensemble of magnetic atoms being aligned by an external magnetic field, and a networked force being given an order to carry out a mission. Just as the magnetic atoms are aligned to a common axis by the applied magnetic field, all the nodes in the network are oriented to a common mission objective. **In a networked force with an operational goal, each node and each link may be considered to have unique values to the organisation**.

By contrast, in most everyday networks each node performs certain functions and serves a purpose of its own, but it would have little or no concern for the existence or objectives of other nodes in the rest of the network, and for that matter for the overall goal (if it exists at all) of the entire network. From a mathematical point of view, most networks in the literature have been treated as homogeneous systems with assumed high degrees of symmetry using an effective-medium approximation. This approach, together with the high symmetry constraints, imposes limitations on one's ability to formulate mathematical models that can capture the nonlocal interactions. On the other hand, in a networked force all the nodes (*e.g*. platforms, units, weapons, ISR[1] systems) and all the links are likely to have significantly different functionalities and capabilities, and thus different values to the organisation as a whole. **In other words, military networks, especially the relatively small ones like the Australian Defence Force, tend to be heterogeneous with a wide ranging variety of functionality and capability across their nodes and links.** Here one may well encounter situations where the severing of many links, or the failure of a highly connected node, would make relatively little impact on the outcome of a mission, whereas the failure of a high value node with few connections could conceivably undermine the entire mission.

Let us now examine the implications of our discussion above using as an example the definition of network connectivity. We begin with Kazunic's well-known expression for the connectivity index C_i:

$$C_i = \frac{k}{N_T(N_T - 1)}$$ **Equation 1**

where k is the number of connections in a network, N_T the total number of nodes. This elegantly simple equation has been derived with the assumption that the network under consideration has a high degree of symmetry, with all the nodes being the same and all the links being identical. On the other hand, if we relax the assumptions and introduce a more realistic expression, we obtain a far more complicated equation of the generalised *Connectivity Measure* [Ling *et. al*. 2005]:

$$C_M(t) = \frac{1}{C^R}\sum_{\mu=1}^{N_T} K_\mu(t) \sum_{\nu=1}^{N_\mu} L^{\mu\nu}(t) \sum_{\gamma=1}^{N_{\mu\nu}} \frac{F_\gamma^{\mu\nu}(t)}{d_\gamma}$$ **Equation 2**

where C^R is the normalisation factor, N_T is the total number of nodes in the network, N_μ is the total number of nodes connected to the node μ, and is equal to $N_T - 1$ if the network connection is 100 percent; $N_{\mu\nu}$ is the total number of possible routes connecting a pair of nodes μ and ν, and is equal to $\{1+ (N_T-2)+ (N_T-2)(N_T-3)+ \cdots\cdots +(N_T-2)(N_T-3) \cdots 2\cdot 1\}$ in a fully connected network; $K_\mu(t)$ is the value of the node μ at time t; $L^{\mu\nu}(t)$ is the value of the link between nodes μ and ν at time t; $0.0 \leq \{K, L\} \leq 1.0$; $F_\gamma^{\mu\nu}(t)$ is the flow coefficient of route γ linking nodes μ and ν at time t, scaled by the graph-theoretic path length d_γ . The flow coefficient, $F_\gamma^{\mu\nu}(t)$, ranges from 0 to 1. Finally, if a route is unidirectional, then we define $0 \leq F_\gamma^{\mu\nu}(t) \leq 1$, and $F_\gamma^{\nu\mu}(t) = 0$.

The normalisation factor C^R is the connectivity measure of a **reference network** with full symmetry (i.e. all nodes and all links have a value of unity) and full connection. It is given by

$$C^R = N_T(N_T - 1)\times\left[1+\frac{(N_T-2)}{2}+\frac{(N_T-2)(N_T-3)}{3}+\cdots+\frac{(N_T-2)(N_T-3)\cdots 2\cdot 1}{N_T-1}\right]$$ **Equation 3**

The benefit of the concept of a reference network is that it enables us to consider a real network with the same number of nodes and same network topology, but which has nodes and links of different values, as **a network with broken symmetries**. While Equation 2 is a useful indication of the extent of symmetry-breaking in a real network, a more powerful application of the connectivity measure C_M is to examine its rate of change as the nodes and links in a network are randomly disabled. This provides a direct measure of the susceptibility of the system to failures (attacks) [Ling *et. al*. 2005]. Work on new symmetry-based methodologies is likely to contribute to a better understanding of the subtle relationships between the underlying

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symmetries in military networks, the range of interactions and the susceptibility of the connectivity measure [Ling 2005].

It is worth stressing that, apart from the reference network and thus the normalisation factor C^R, no assumptions have been made about the symmetry of the network as described by Equation 2, as every term inside the summation signs is allowed to be different from all others, including the directionality of the links. This means that the equation represents networks that can at times be arbitrarily asymmetric with some nodes and/or links having a far higher value to the mission success than others. The time-dependence of Equation 2 also means that the values of each node and each link will vary with the changing demands of the mission and circumstances. An interesting consequence of this is that, while high value nodes and links are obvious targets, the duration of the risk may be relatively short. For example, in an anti-ship missile defence scenario involving an air warfare destroyer and an airborne early-warning aircraft, it is clear that the links between the air and maritime assets are of vital importance. However, once the threat has been neutralised, the value of such links would be greatly diminished.

NCW MODELLING AND SIMULATION

Due to the complexity of network dynamics and networked operations, simulation is often the only means for analysing NCW and EBO problems, in particular for assessing the less tangible issues such as the value of information, the agility of a force or the projected effectiveness. The key challenge is to ensure that the mathematics that underpins the modelling and simulation can indeed capture the fundamentals of the problem at hand, such as the ranges of interaction, the degree of symmetry and the dynamic nature of military operation.

Over the past two years the Australian Department of Defence has funded the development of a modelling and simulation tool called the Dynamic Agents Representation of Networks of Systems (DARNOS) for NCW modelling and analysis. DARNOS begins by modelling a networked force as an organisation with common goals shared by all its members. It employs a new organisation-oriented agent technology, in which a top-down approach can be taken to model the organisational structure and interactions of the entities. The benefit of this approach is that each agent (entity) has a perception of the existence and role of both itself and the others in the organisation, with the result that it is now possible to model not only how each entity functions on its own but also how it operates in relation to other members of the same organisation. As its name indicates, DARNOS allows one to model the dynamic changes of networking and C2 structures, as well as the time evolution of single and multiple networks.

When coupled to a simulation environment with a battle-space representation, such as the BattleModel developed in DSTO, DARNOS has the unique capability of modelling the dynamics of networked operations from information-gathering through a network of spatially distributed ISR assets to decision-making, and from the dissemination of decisions (issuing of orders) to the execution of physical actions in space and time, all in one single M&S package. In short, the DARNOS-BattleModel package has been explicitly designed for analysing the impact of different C2 and networking structures on the operational effectiveness of a networked force; **that is, for examining the force's ability to conduct Network Centric Warfare as stated at the beginning of the Introduction section.**

Finally, one of the great challenges of understanding complex systems is the ability to identify the origins of unexpected outcomes of the simulation. The DARNOS-BattleModel package provides complete traceability in the simulation so that every event at every step of the simulation, from ISR inputs to decision-making, from dissemination of decisions to actions in space and time, can be scrutinised and their effects on the final outcomes emerging from the simulation can be properly understood.

SUMMARY

The central theme of this work is that complex systems such as networked forces can be highly heterogeneous in functionality and capability (*i.e.* their values to the organisation or the force) with many types and ranges of interactions. The underlying symmetries and the nonlocal nature of the interactions in these systems are closely interrelated and could have profound implications on our ability to analyse the benefits of the concepts of NCW and EBO, and to identify the optimal ways of implementing them. Due to their complexity, simulation is often the only means for analysing these problems. A challenge for us is to ensure that the mathematics underpinning the modelling and simulation have the requisite level of modelling fidelity of the interactions in these systems, and that the modelling is consistent with the nature of the problem. Finally, the need to include in our modelling and simulation the dynamic interactions in complex systems cannot be overstressed. Efforts to model and simulate the dynamics of a complex system should include the ability to track the events so that any *emergent* outcomes of the simulation can be traced and correctly interpreted.

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