"The Influence of Paradigm Shifts on the Practice, and the Perceptions, of OR" by John Dockery, PhD
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ABSTRACT

The author draws upon his extensive experience with DoD to present his case for the impact of OR on the conduct of military operations. He finds the contribution to have been very, very uneven. His insights reflect the period through about 2000 as he retired in 1999. Some comments, dealing with scattered exposure to Homeland Defense, are also included. The impression, summarized as a serious lack of impact, is advanced. This observation is discussed in the context of Kuhn & Tucker's work on paradigm shifts. "Why is it that the practitioners by and large tend to be starting from scratch?" asks the author. Possible reasons for lack of expected impact are advanced. Chief among them is the way OR is perceived within the military coupled with the impact of security.

THE PRIMARY THESIS

It is this author's contention that OR has not had the dramatic influence on the military that it might have had, especially in areas with new challenges. Drawing on his experience the absence of impact is attributed to what is known as paradigm shift that was occurring within the mathematics community. (Such a shift is a kind of culture clash of which more will be said later.) That clash, it is contended, spilled over into the military OR community with deleterious effects. It is further contended that the effect was highly non-uniform. The very nature of the military society further amplified the centrifugal tendencies accompanying the fight over the most fundamental precepts in mathematics. The battle was most bitter over a discipline called fuzzy set theory.

SOME NECESSARY BACKGROUND

Because this presentation contains a high quotient of opinion, the audience deserves to know something of author's background. Figure 1 highlights of the author's background are summarized in broad terms.

LOCATION	YEARS OF EXPERIENCE	MANAGER	ANALYST
Ass't Vice Chief of Staff, Army	5	√	√
Joint Staff	8	√	√
SHAPE Technical Center	4	√	√
(Army) Concept Analysis Agency	7		√
Defense Information Systems Agency	7	√	√
Brookings Institution	1	√	
George Mason University School of Public Policy	4	(√)	√

Figure 1: Author's Career Background as It Pertains to the Practice of OR

A self characterization is that the author, in addition to his day job, operated as a kind of "talent scout" for new approaches. Basically, he functioned as both a scientist and a manger/advisor, often simultaneously. A summary (Figure 1) lists almost 15 years in policy making branches of DoD. (The agencies were the AVCSA (Ass't Vice Chief of Staff, Army) and the Joint Staff plus a year at the Brookings Institution). Another four year sojourn was an assignment to the SHAPE Technical Center (STC) in The Hague, Netherlands. Finally, I must in include the years with analysis organizations. CAA (Concepts Analysis Agency) and DISA (Defense Information Systems Agency). During all those years I often found myself in the rather incestuous position of both producing and consuming the results of OR.

I now turn to a consideration of the state of the art in science during the 30 or so years in which I was involved with military science. I intend to present a number of case histories for which only a bit of doggerel does them justice: to wit

Of all the words of tongue and pen
The saddest of them all is
It might have been

Figure 2: A Bit of Applicable Doggerel

In my experience military operations research stood outside the normal paradigms of science. (Logistics for reasons to be elucidated elsewhere is not included in the sorry tales to follow)

THE SCIENTIFIC PARADIGM

Circa 1962, Thomas Kuhn published a book entitled *The Structure of Scientific Revolution.* [6] In it he describes the nature of the scientific approach in terms of a paradigm. (We may regard a paradigm as a structured way of reasoning about something.) According to Kuhn, the scientific paradigm is the way we ordinarily do business. Figure 3 lists the four attributes of the scientific paradigm.

1. What is to be observed and scrutinized.
2. The kind of questions that are supposed to be asked and probed for answers in relation to this subject.
3. How these questions are to be structured,
4. How the results of scientific investigations should be interpreted.

Figure 3: Attributes of the Scientific Paradigm

Where is the revolution in the foregoing paradigm? There is none until a paradigm shift occurs! What exactly is a paradigm? Why does it evince such bitter recriminations? Kuhn argued that during periods of "normal science" scientist's work within the same paradigm. Scientific communication and work proceeds relatively smoothly until anomalies occur, or a new theory or model is proposed, which requires understanding traditional scientific concepts in new ways. The new theory rejects old assumptions and replaces them with new ones. A remark by Kuhn is instructive on this point. Sometimes the convincing force is just time itself and the human toll it takes, Kuhn pointed out, using a quote from Max Planck: "A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it." Even death may not be the final arbiter as in the case of Bayesian statistics. Figure 4 summarizes the process.

1. Everybody is fat, dumb, and happy. There are no anomalies in the data or theory, *nor could there be any.*
2. Anomalies creep into the data and theory, but they are dismissed.
3. Anomalies can no longer be ignored, but the establishment digs in its heels.
4. Two paradigms now co-exist, one traditional and another new. The paradigms are incommensurable. Likewise, are the concepts used to understand and explain basic facts and beliefs?
5. Eventually, but not always, the old gives way to the new.

Figure 4: Steps in a paradigm shift

SETTING THE STAGE

Initially OR had at least two major successes. One was in the area of logistics and manufacturing. The other was on the subject of nuclear exchange. Logistics benefited from the growth of linear programming (LPs) and the availability of computers. The mathematics behind LPs was sufficiently mature to lead the applications sector. The implications of nuclear warfare drove the development of game theory. An exhaustive mapping of nuclear outcomes was possible. And, it was done. Leaders in the field of mathematics did the heavy lifting. Both topics could be related to accepted principles of mathematics by the leadership.

WINNERS	EXAMPLES	USERS
Linear & Non-Linear Programming	Traveling Salesman Problem	Logistics & Manufacturing
Game Theory	Zero Sum Games	Nuclear Exchange
Graph Theory	Packing Circuits onto Chips	Electronic Circuits

Figure 5: Winners in the Impact Game

But, more complex challenges awaited the OR community. These would involve questions at the root of mathematics such as imprecision and order and dimensionality. These were items so long established that they were considered inviolate. Pause a moment and consider the relationship of the physical sciences to mathematics. Until the last century mathematical discoveries ran far ahead of the physical sciences, sometimes by centuries. When a branch of mathematics was required, it was, so to speak, already in the medicine chest. Enough time would have elapsed for there to be no doubt about the mathematics needed. Some time after, say 1950, the medicine chest became bare. Nor was this the only change rocking the scientific community. Computer usage was redirecting mathematics development away from closed form solutions toward problems involving finite arithmetic. Some examples of bedrock mathematical assumptions overturned in the last 50+ years.

INVIOLATE PRINCIPLE	REAL SITUATION	CONSEQUENCE	NEW DISCIPLINE
Dimensions Are Integers	Dimensions Can Be Fractions	Boundaries are Longer (Shorter) Than We Knew	Fractal Geometry
Set Membership is Either One or Zero	Set Membership Can Now Be Partial	*Mathematics Incorporates Imprecision *Logic Becomes Multi-Valued	*Fuzzy Set Theory *Possibility Theory
The World is Linear	The world is Very Non-Linear	More Often Than Not, ‘C’ follows ‘A’	Chaos Theory
Self-organization is Not Possible!	Self-organization is Key to Understanding the Existence of Large Scale Systems	*What Good is Planning *Feedforward Principles Established	*Ecology *General Systems Theory

Figure 6: Examples of Bedrock Assumptions Overturned

The educational system did not prepare either the literate laymen, or those who thought they knew mathematics, for this kind of bump in the road. What was, perhaps, more disconcerting was that serious computations could be done with a handful of simple relationships.

Did anyone see this coming? Only one name comes to mind, Ross Ashby, who died in 1972. By trade, he was a psychiatrist. Although a founding father of cybernetics he has already been largely forgotten despite having formulated at least three laws pertaining to large systems:

1. Homeostasis
2. Law of Requisite Variety: That which is controlling must reflect the complexity of that which is controlled
3. Principle of self-organization

The case for, and against, a number of the paradigm shifting applications proposed was not always helped by the labels assigned to them. Tell a military officer, proud of his exact command and control equipment, that you are using fuzzy sets to analyze his decision making, and you have a very nervous man. The same observation applies to catastrophe theory; or better yet, chaos theory.

Now put yourself in the role of program manager. Fuzzy Set theory is proposed for the approach to a vexing problem in decision theory involving ill defined predicates and bad data. The program leader has never heard of the technique, but is willing to learn, so he follows his own paradigm. That paradigm says ask a friend and/or an expert. What is he to do when the expert tells him that the selected approach is all smoke and mirrors? Consulting several experts, he hears from all points of view. They range from scorn to almost rabid advocacy. Not wishing to jeopardize his program, he upbraids the people who proposed the fuzzy sets as proposing a solution that is too complicated. Alternatives are accepted. The fact that the computations may be much more complicated than the paradigm shifting approach does not matter. Others couch their refusal in somewhat different ways by demanding the solution use techniques, which are familiar to the manager.

Sometimes the response is a retreat to an earlier paradigm, but one which is now dressed up in a complex computer program. One such example centers on Lanchester equation(s). If one were to look for examples of paradigm shifts, one might find oneself in the following situation. Instead of finding a list that went back weeks or even years, the list would encompass millennia. Such a list might read as follows in Figure 7.

Sun-Tzu
Van Clausewitz
Lanchester
Creveld

Figure 7: Shifts in Military Analysis

Lanchester was trying to understand air warfare. He concentrated on the duel, for which the ordinary differential equation(s) (ODE) he used sufficed as a starting point. The next logical step would have been the use of partial differential equations. This did not occur. The excuse was always this. Computers were not fast enough, nor were they powerful enough. That may have been true in the beginning, but, as each year passed, it became less true. Santoro and Protopopescu at the Oak Ridge National Laboratory applied the lessons of nuclear modeling to a set of diffusion equations. (PDE) [8] Using data from a national test range, they were able to reproduce the flow of combat. Although this was published in *Signal*, it was ignored by the modeling community. [3] At the next Lanchester equation meeting they were back to diddling the basic ODE. Worse yet was ground force analysis based on computer simulation. The simulations buried a number of Lanchester Equations deep within the reigning computer programs. Models were then *tuned* by means of input data manipulation until plausible results were obtained.

How bad was the situation, which this paper alleges. To give some estimate, it becomes necessary to scale up the problem. We might ask: "What happens when there are 'n' paradigm busting subjects to hand some of which are, in fact misleading, if not wrong". The answer is that progress stops. People hang onto what they know. Sometimes they will even retreat to previous positions. Add to this mix a group of onlookers, for whom their academic training did not equip them to choose sides. Yet, the observers had a job to be done. And so a decision was required! And so a decision was made! It did not include paradigm breaking mathematics.

We are now going to present some thumbnail case histories of missed opportunities. We begin with fuzzy set theory. Others will include elements of chaos theory and catastrophe theory. We will conclude with possible lessons for Homeland Security.

CASE HISTORY NUMBER ONE—FUZZY SETS

We have already said that paradigm breaking research is often fought with utmost vigor by the establishment. This is seen in the extreme in the case of fuzzy set theory. A brief introduction on fuzzy sets may help in an understanding of what follows. (See Sidebar)

Fuzzy set theory first made it appearance about 1965 in a paper in *Management Science* entitled "Decision Making in a Fuzzy Environment". [2] Richard Bellman and Lotfi Zadeh were the authors. Lotfi himself was an Iranian by birth but as the son of diplomats, he grew up in Tashkent, USSR. The term *fuzzy* was the choice of Zadeh. While quite accurate, technically speaking, *fuzzy* carried a heavy load of adverse connotation. It

> Fuzzy set theory is based on a simple relaxation of the rules governing set membership. The conventional set has something called a *characteristic function (x) where f(x) = 1, $x \in S$, and f(s) = 0, $x \notin S$. With the characteristic*

also carries reference to childhood. It is an insult to tell someone that he is a fuzzy thinker, or that his approach to a problem is fuzzy. At the time of the paper, Zadeh was in his forties at Berkeley, and comfortably ensconced as a professor of systems and computing.

At the time, I had just joined the AVCSA office from the aerospace industry, which was about to go into a decline that has lasted until now. A few months exposure to the Pentagon had convinced me that it was indeed the quintessential fuzzy decision making environment. I saw a match between the work of Bellman and Zadeh, and the task within the Pentagon. For the record, the specific problem at hand was budget generation using data that was historical (being a record of the previous year) to ten years out. Though the latter were at best SWAGs, they were treated as if they were real. I believed that fuzzy accounting was the answer since current costs and projected costs, while considered as real numbers, were not. To introduce fuzzy numbers into a budget drill was heresy.

Needless to say, I was given another task!

In Japan a statue would be erected in Zadeh's honor. While back in this country, it is said that the editor of the most prestigious mathematics journal had issued standing orders that anything submitted with fuzzy in the title was to be returned unread.

function now renamed the 'membership function", defines a fuzzy set as a set of ordered pairs

> $\{(x),\mu_A(x)|x\varepsilon X\}$ where x is a collection of objects in X, and A is the fuzzy set in X

This does not look like much of a change, but it is profound!
Numbers as we know them from arithmetic are no longer the same, being now defined no longer on a field, but rather on a ring. Conceptually the results are stunning. Fuzzy sets incorporate imprecision in a rigorous manner. The introduction of the membership function now represents not only YES and NO, but all the MAYBEs in between.

What Zadeh had done, mathematically speaking, was very simple, but the consequences are still rippling through the structure of mathematics. He had introduced a multi-valued logic. In fuzzy logic the answers can be any value on the closed interval [0,1]. The membership function now represented the degree of membership in a set. It developed that the mathematics associated with fuzzy sets was very straightforward. Any engineer could do serious computation. It became one of those rare instances in mathematics where the computation outran the theory. It was not long before engineers began building fuzzy processing chips. Outside the USA packaging material trumpets the fact that the device inside uses fuzzy logic. Inside the USA nary a word. Yet, for example, all cameras with stability control utilize fuzzy circuits. Undergraduate textbooks now have chapters on fuzzy sets, but to the public at large the subject does not exist.

CASE HISTORY NUMBER 2—CHAOS & CATASTROPHE THEORY

Although both of these disciplines deserves its own section I have linked them because they provide different perspectives on the same class of problem; or so I believe. Each is treated in turn.

After failing to get initial traction, ***chaos theory*** seems at last to be getting some well deserved press. Military experts, especially those in the command and control end of the business, still are not comfortable with ideas of chaos theory. One way to sum things up is by asserting that order exists in the presence of disorder. Chaos theory is quite useful in dealing with the political aspects often associated with military problems. Two situations may look quite similar but they may reside in two distinct "basins of attraction". (See sidebar). One of these basins might represent a stable regime while the other represents a destabilized regime

The first breakthrough came when some bored scientists were killing time in an airport. Playing with a portable computer, they discovered an odd property of the logistics equation when iterated.

Let $x_{n+1} = a\, x_n\, (1 - x_n)$.

Below a specific value for *a,* the iteration always give the same value for the iterate. But for values of *a* larger than this value, the value of the iterate begins to behave randomly! It does not matter how slight the change may be.

That such complex behaviour could lay hidden in such a simple relationship caught the attention of computer specialist. It was no longer safe to iterate endlessly as was common place in certain force structure simulations.

Chaos theory stands to make its greatest contribution in helping to understand and to mathematically describe combat. A colleague and I have shown that while the commander can not control all the details of combat, he can control the boundaries of the chaotic region of phase space. If you think about it for awhile, you can also get an appreciation for the role of chaos when forces are evenly matched.

We turn now to ***catastrophe*** theory. (*See sidebar*)
At first, it seemed a wild success. People were observed to be reading inscrutable texts on the subject on buses on the way to work. It promised not only an understanding of physical problems, but social and political problems as well.

Using catastrophe theory without tears takes both imagination and a willingness to stand conventional problem solving on its head. First, you assume that the problem probably has a differential description even though you have no idea what that description may be. For, if this assumption is true, you know that it is confined to a surface, which you can generate.

The language of ***chaos*** uses the term "attractor" to describe how a mathematically described solution is pulled toward a final state. The simplest are "point attractors". When such exist, no matter what the initial conditions may be, the system always ends up in the same place. Think of a marble poised on the rim of a bowl. If it is flicked off the rim to the inside, it will always end up in the same place no matter where it starts, and with what initial conditions! This is an example of a basin of attraction!

Not all attractors are so neatly described. At the other end of the spectrum are so called chaotic attractors. They usually have some footprint (in a suitable phase space). No matter what the footprint may look like it divides the phase space into allowed and forbidden regions. Random events can kick a solution from one basin of attraction. Chaos theory has proven quite useful in ecological modeling. An early success story modeled the spruce budworm infestation in the forests of Northern Canada. I believe it was the first to show the importance of forest fires in the overall scheme of things.

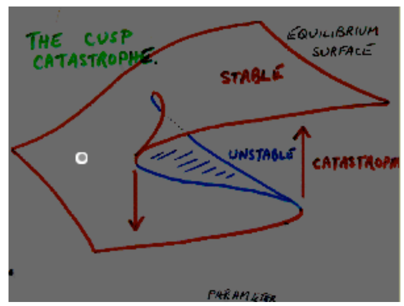
You can also speculate on the local behaviour of the surface. One (or two) of the axes is (are) known as the "output variable(s)". The others are called the "control variables". In the example of the CUSP, we see that the local geometry splits the plane into two sheets. The control variables move you about the upper and lower sheets. Should you come to the edge, and fall off, there is no immediate way back! Thom labeled this a catastrophe. An instructive example is a ship capsizing. Treating the control variables parametrically, we can use the CUSP for all manner of sketching.

This works for social, economic, medical, political, … problems providing insight to a whole class of problems. Frustration set in when the images were pushed too far too fast; and when it became clear that statistical fitting programs were non-trivial. It might be noted that the physics community suffered no such frustration. They had physical cases of the higher order catastrophes, which fit the theory nicely.

Before closing this brief excursion, a word or two is in order concerning the work of Prigogine on the thermodynamics of systems far from equilibrium. They are typically high dimensional systems However, such systems go through phases in which only a few variables are important. This can be modeled on high order catastrophe surfaces when a control variable is unclamped.

One other intriguing thing about catastrophe theory is the existence of the 10 space- and time-dependent cusps. They show respectfully: [time] describe events at a *fixed point in time at different spatial locations*; while [space] describe events *at fixed spatial locations as time progresses.* [4] Woodcock and I believe that this duality permits the development of models in which combat, and command and control, can be traded off. Remember! the classical defense trades space for time; while the classical offense trades time for space.

Catastrophe theory is due to Rene Thom. His contribution was the demonstration that ***all*** the solutions to certain differential equations lay on one of 7 surfaces All but the first two, known as the as the FOLD and the CUSP, are higher than 3-dimensions. What is conventionally plotted are the projections into three space. We illustrate some features of catastrophe theory with the CUSP. A thumb-nail sketch is shown below.



The general equation of the cusp is

$$x^3 + ax + b = 0$$

Higher order surfaces have fanciful names like swallowtail, butterfly, and umbilic. Dockery and Woodcock have been able to dig out a generalized Lanchester Equation from the Double Cusp, which is NOT one of the 7 elementary equations, and has 10 dimensions! The break-down shows 3 control each for x and y; and 4 *shared* control variables

CASE HISTORY NUMBER 3—SIMPLICIAL COMPLEXES

This tale really is about something called q-analysis methodology. The q comes from its use to designate a degree of dimensionality to a structure; and also its degree of connectedness. The modern army is all about structure, yet this tool, which seems the quintessence of structure, apparently goes unused. It also seems ideal for Homeland Security analysis. Where two or more structure conflict (or combine) it is possible to build what are known as *star-hub* structures. The bridge between two star-hubs is known as a *March prism* . The prism shows what aspects of the individual structures interact. This work is due to Jeffery Johnson. [5]

Q-analysis was originated by Ron Aitkin; and published in a book entitled *Mathematical Structure in Human Affairs.* [1] What Aitkin did, was to take two sets of objects, and build a matrix of ones and zeros. Each set served as the labels for one of the two axes of the matrix. If the object in set 1 was related to an object in set 2, a one was placed in the matrix. If no relationship existed, then a zero was entered. It doesn't seem like much, but it is, mathematically speaking, quite deep. The resulting matrix is known as a simplicial complex.

It is a very practical way to glean information from a poorly structured set. It happens very often that while binary relationships are known, the global picture is absent. What the q-analysis methodology does is to erect temporary scaffolding, which in turn permits inferences to be collected about the global situation. It quite literally bootstraps from local to global. It was shown early on that the q-analysis methodology was isomorphic to graph theory. This led detractors to inquire why graph theory was not used instead.

What many failed to understand is that q-analysis was useful when you did not have enough knowledge to apply graph theory. It develops that a simplicial complex is a kind of crossover point for several kinds of mathematics. The one of most interest is the concept of a backcloth. In direct analogy to the term as used in the theatre, a backcloth represents the skeleton of the landscape over which the traffic must flow. Traffic means anything flowing around or through the structure. Changes to a structure show up in changes to the matrix. By looking at the geometrical rendition of q, the change to a structure in q-analysis shows whether it is local or global.

From its inception the reception of q-analysis has been chilly. In the 1990's a number of papers on the homotopy of simplicial complexes established q-analysis in its own right. In summary, starting with no more than a matrix of ones and zeros, it is possible to build n-dimensional shapes. Where difficulties in traffic would occur, they show up as "holes" in the structure. Of course, you cannot see the structure. Oh well! No one said it would be easy.

CASE HISTORY NUMBER 4—SEMIOTICS AND CYBERNETICS

Sometimes resistance to the paradigm shift is defensible. Take two examples. The first is cybernetics. It was touted as the advent of machine intelligence. Unfortunately, it became associated with Soviet research during the cold war. OR work that was entangled in the controversy surrounding cybernetics usually lost out. In the West an alternative existed. This was the discipline of artificial intelligence. It eventually absorbed cybernetics.

Sometimes it is your scientific zip code that drags you into the fray. The best example of with which I am familiar is semiotics. This is the study of syntax and meaning. Unfortunately, semiotics acquired a kind of *new age* patina. It attracted camp followers who earnestly believed that crystals had curative powers. Too many of the adherents to semiotics were more of a philosophical bent. Too much of semiotics was worthless. Nonetheless, in the right hands, it appears to offer insights not available anywhere else. Two scientists, who are doing serious work in the field of semiotics are Joseph Gougen at the university of California at San Diego, and Cliff Joslyn at the Los Alamos National Laboratory. Gougen has used category theory to sharpen the definitions associated with semiotics. It appears to me that Homeland Security, which must deal with syntax and the like, as if they were contact sports, would benefit from the emerging insights of semiotics.

LARGER PARADIGMS

More encompassing paradigms were also emerging. Two that affected the practice of OR come to mind. One is the recognition that much in nature is self-organizing, including combat. Acceptance of this principle probably cost a generation of analysts. The other is the concept of agent based simulation. This was especially pioneered at the Santa Fe Institute in Santa Fe, NM. They developed a framework called SWARM. With agent based simulation ones gives up a high degree of control.

The analyst knows that he will never see the detail he once expected. He also must believe that the myriad possible interactions that occur in an agent based simulation represents ground truth.

MARK TILDEN AND HIS ROBOTS

As a postscript to the focus on the methodological aspects of OR, I am reminded of the story of Mark Tilden and his robotic critters. An excellent synopsis of Tilden's work, and his accomplishments, can be found in the *Smithsonian Magazine* for *February 2000.* [9] It brings a somewhat bizarre twist to the issue of paradigm shift. In a digital world, Tilden has rediscovered analog computing. Eschewing the machine with a digital brain, i.e. a conventional computer, he builds devices, which program themselves. For example, a device will change its mode of locomotion when moving from sand to mud. The mathematical model for this is quite complex involving motion on a surface called a toroid. He has designed some to look for mines. They are cheap and effective. The community response might be characterized as studied indifference. The challenge for OR is to be certain they are tested against the problem, and not against competing digital devices,

LESSONS FOR HOMELAND SECURITY

Based on the unclassified, largely methodological material, which I have seen, I fear that Homeland Security has learned very little from DoD. Speakers present material, with which they are familiar, but which has little connection with the real problems. There also seems to be a preoccupation with incomplete or questionable data bases. Techniques exist to draw as much information as possible from data bases, which are corrupted. The power of the simplicial complex also seems not to have been tapped. Remember, one can construct a complex, expressed as a matrix of ones and zeros based only on the known, or suspected, link between any two items in the sets containing the elements of the two axes of the matrix.

SUMMARY

The purpose of this paper has been to explore what the author perceived as a near shutdown in positive impact upon the practice of OR during the last third of the last century. This is attributed to a series of paradigm shifts in the supporting mathematical community. A number of thumbnail case histories are offered in support of the author's thesis. The author cautions that this piece is a highly personal reaction to a perceived situation. In Figure 8, which follows, is taxonomy of sorts, which illustrate the various areas in which he has worked. The columns labeled APPLICATIONS & USERS. reveal a layering in the application of OR to military problems. This is probably worthy of further investigation.

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GROUPINGS	SUB-DISCIPLINE	APPLICATIONS AREA	POTENTIAL USER
FUNDAMENTAL MATHEMATICS	Fuzzy Sets	Decision Theory Control Theory	Management
	Catastrophe Theory	Global Strategy	Designer
	Chaos Theory-	Prediction	Weather Forecaster
	Simplicial Complexes	Ill-structured Relationships	Sociologist
	Fractals	Geography	Scene Designer
	Category Theory	Theoretical Mathematics	Mathematician
	Cellular Automata	Modeling	Games Designer
	PDEs	Diffusional Flow Computations	Aircraft Designer
DERIVATIVE	Neural Nets	Draws on two or more of the foregoing (List is decision theory oriented)	OR Team
	Possibility Theory		
	Fuzzy Conditionals		
	Belief Theory		
	Genetic Algorithms		
WORLD VIEWS	Cybernetics	Organizes material in relation to other projects for the manager	
	Semiotics		
	Virtual Reality		
SYSTEMS THEORY	Self-Organization Principles	Source of Governing Principles Behind the World View	
	Dynamical Systems Theory		

Figure 8: Author's Own Taxonomy

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