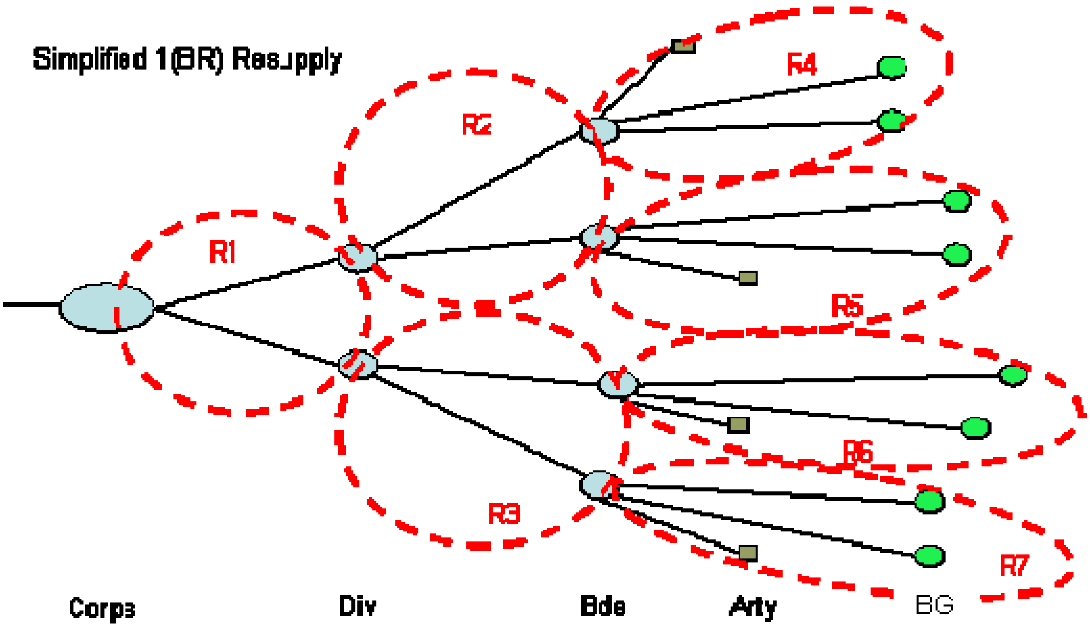
The Use of Linear Programming in Military Operational Analysis

This paper presents a personal review of the use of Linear Programming (LP) techniques in Military Operational Analysis studies over the period 1967 – 2008. Applications described include Logistics Modelling, Reinforcement and Deployment modelling, Air Defence Balance of Investment (BoI) Analysis, and Cross-capability BoI Studies. In particular, it emphasises the use of LP to explore the solution space and to identify worthwhile areas for more detailed simulation modelling.

<u>Logistic Modelling (1968-9)</u>

1. My first use of Linear Programming was for a study designed to assess whether the logistic transport in support of 1BR Corps was sufficient to sustain combat elements in general war on the NATO central front. Likely consumption rates for POL and ammunition, battle duration and unit movements were drawn from a computerised wargame known as the Central Front Game. The number, type and organisation of the logistic vehicles and the planned use of depots, dumps and resupply points were known. The classic OA approach at this point would have been to use simulation to model the movement of stocks through the system to meet the demands of front line forces. However, there were two reasons for not following this course. Firstly, no such simulation existed and in the time available it was not going to be possible to develop one. But perhaps more importantly, the output of any simulation would have depended on the degree of efficiency with which the analysts could balance the transport assets across the resupply system and over time. This would be likely to vary significantly from run to run, resulting in inconsistency and possibly incoherence in the outputs. A LP approach offered two distinct benefits. Firstly, once the problem was understood, the model could be formulated in a few days and fed into a standard package. Secondly, the model would be capable of exploring the whole solution space and delivering a consistently good result over a wider range of variations.



2. The resupply system was represented as a series of nodes and linking routes that mirrored the command chain from Corps to Battlegroup. The operation was divided

into a number of time phases. Within each phase, distances remained constant. The links were assigned to "transport regions" and the numbers of trucks and tankers assigned to each region in each time phase was a study variable. Each front line unit and resupply point in the system had a starting stock and a required stock level. If the stock fell below this required level a shortfall was logged. The objective function of the model was to minimise the maximum shortfall across the system (MINIMAX). The model proved to be flexible and fast running, showing the expected levels of shortfalls over time across the system for different assumptions on movement patterns and consumption rates. It also enabled advice to be given to the logistic managers on the balance of transport between the different resupply regions. The downside, from a personal point of view, was that in 1968 the MoD did not possess a computer capable of solving a large LP. I therefore found myself spending several evenings a week with a Central London company using their CDC6600 machine.

3. The model was used in a follow-on study that tested the ability of Warsaw Pact forces to sustain the advance rates that were predicted for them. In particular, there was interest in the numbers of trucks that would need to be mobilised from the civil economy to provide the necessary support, providing key indicators and warning.

<u>Reinforcement and Redeployment Modelling (1969-75)</u>

4. In 1966 the "Intervention and Amphibious Working Party" (IAWP) study set out to determine the most economic mix of air and sea transport, and of pre-positioned forces and equipment to meet worldwide requirements for deployment of forces to prevent or deal with a range of threats. During the study the government of the day decided to withdraw from "East of Suez" and the study was refocused on the need to reinforce and redeploy forces in the NATO Central Region and its Northern and Southern flanks (the Reinforcement and Redeployment Working Party (RRWP)). A large LP model was developed to solve these problems. Because the mix involved stockpiles that were either established or not, a mixed integer formulation was adopted, with the run-time penalties that this imposed. It was run on a Univac 1108 machine owned by BP. The model was successfully used to explore a range of options and to generate the evidence that underpinned the Working Party's report. The model and data were archived in the form of boxes of IBM punch cards stored in the corner of a hut.

5. In about 1970 I was asked to revisit this analysis to answer some questions relating to retention and procurement of air transport aircraft. The stored punch cards had suffered from damp and many were unreadable, but I managed to identify a stored basis from which the model could be restarted. I was then able to vary the constraints on the maximum and minimum numbers of the different aircraft types included in the mix and to see how the remainder of the asset numbers varied and the impact on cost, including the foreign exchange component, which was a particular issue at the time. This work resulted in the cancellation of the option to buy C5 aircraft and to the early retirement of the Brittania fleet.

6. In 1972 reinforcement and redeployment capability again became the focus of attention, this time because there were concerns that under the existing plans, forces were arriving far too late. The movement of force elements into Europe was the subject of a series of reinforcement plans that were to be implemented sequentially. This resulted in a total reinforcement time of 42 days, with significant inefficiency in the use of transport assets. We were charged with a study to determine how reinforcement times could be minimised.

7. I was asked to lead this work, and immediately identified two problems. Firstly, the original model had been directly coded in MPS format, with the coefficients calculated by hand, and with no record of those calculations. It would thus be difficult to reconstruct the model with different parameters. Secondly, the original work had identified the least cost mix of assets, including stockpiles, to deploy a given set of forces in a given time. We now needed to optimise on time, but since time was a factor in many of the coefficients, to do this directly would result in a severely non-linear formulation that could not be solved.

8. The solution to the first problem was to build a matrix generator that would take in the basic data and assumptions on loads to be moved, geography and the characteristics of the deployment assets, and would process them to generate the matrix in MPS format for input to the LP package. To achieve this, Martin Beale of Scicon was brought in as a consultant with considerable experience of developing similar software. With his help we were able to develop the required functionality and compile a data set for the new model within about three months.[1]

9. The second problem was solved by making use of a standard feature of LP packages. The loads to be moved were expressed in the right-hand sides (RHS) of a number of the constraints in the model. We set up the model with an estimate of the shortest time in which the movement could be achieved and with the RHS values for the loads set to half of their actual values. We then used the RHS parametric function[2] to increase these values from x0.5 to x1.5 in small steps. At some point in this process the model would become infeasible. If this happened close to x1, then the time estimate was a good one. If infeasibility occurred below x1 then the estimate was too low, if above x1 then too high. This allowed the optimal time to be determined to within 5% in two or three iterations.

10. This study, which became known as the Concurrency Study, demonstrated that, by implementing the different reinforcement plans concurrently, and by making full use of the available transport assets, ports and airfields, the deployment time could be reduced from 42 days to 23 days with no additional investment.

<u>Air Defence Mix Study (1982-3)</u>[3]

11. In 1982 I was invited to take over a study that aimed to determine the level of investment in ground-based air defence (GBAD) needed to protect 1(Br)Corps and the optimum balance between area and point defence systems. The study had identified likely tracks for attacking aircraft, making best use of terrain screening, to attack identified targets on the ground, both deployed force elements and key infrastructure. It had also identified potential GBAD sites giving good coverage to the deployed force elements and infrastructure. A simulation (called PARADE) had been developed that could represent a number of air raids against a variety of targets over a 24 hour period. The study team had identified a number of possible mixes of Rapier and HVM at different investment levels for assessment in the PARADE model. The measure of effectiveness was the number of enemy aircraft successfully delivering their weapons against friendly ground forces.

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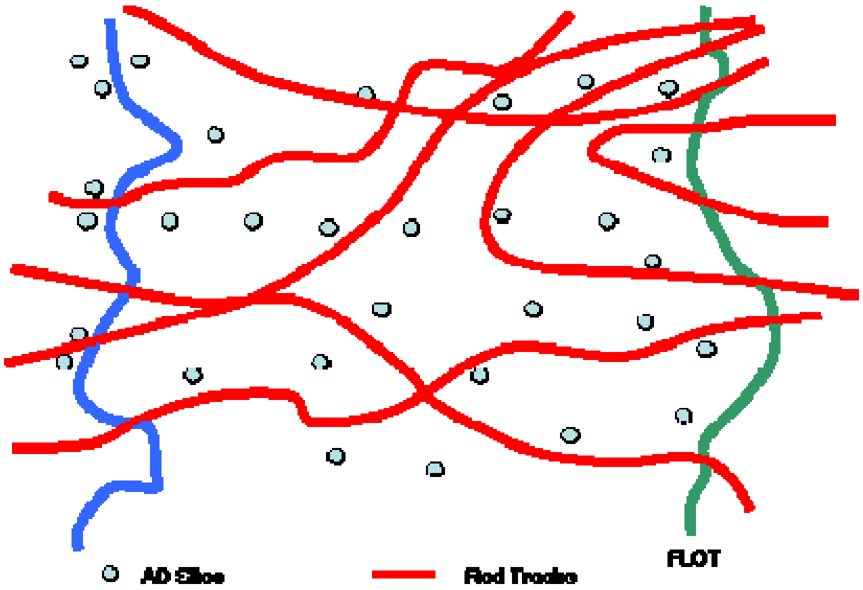
[1] "Mathematical Programming in Theory and Practice" ed. P L Hammer and G Zoutendijk, North Holland Publishing Co., 1974.

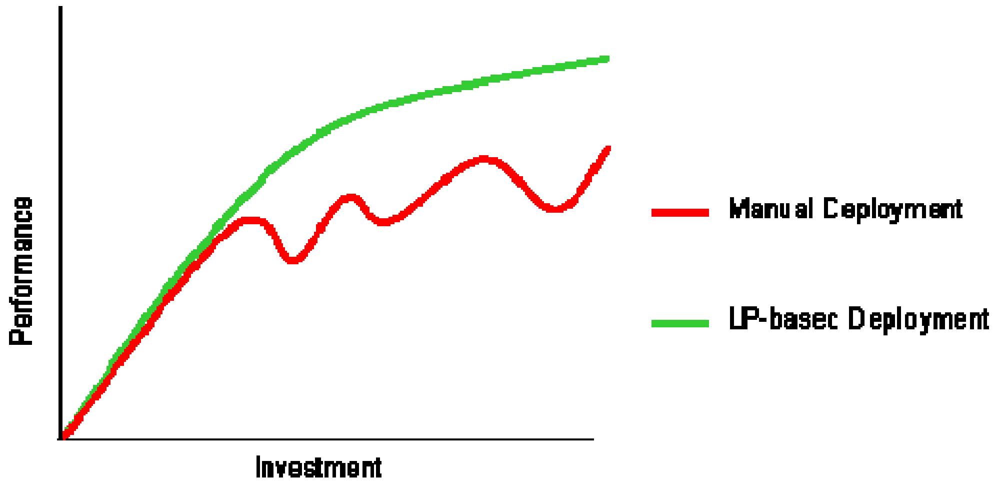
[2] RHS parametrics allows the package to solve the problem at the initial value and then to vary the selected set of RHS parametrically from an initial set of values to a final set whilst maintaining an optimal solution.

[3] This work was the subject of a paper at 3ISMOR in 1986.



11. When the different options were run through the simulation, the results were anomalous, with increases in investment sometimes resulting in reduced effectiveness and different combinations of attack routes resulting in different patterns of performance. The problem was that the deployment of air defence systems for the different investment levels and balances between the two system types had been carried out by a military officer who had probably done a good job with his initial lay-down, but who was unable to maintain the quality of the allocation of systems to sites consistently across all of the variations. The solution to the problem was to build a simple LP model of the allocation process.

12. To enable this, the PARADE model was adapted to generate data on the potential number of kills achieved by each system on each site against each track. A mixed integer formulation was used that allocated GBAD systems to sites up to a given level of investment in such a way that the potential number of kills against the worst track was maximised (MAXIMIN). By varying the cost constraint using RHS parametrics it was possible to track, in a single run of the model, how the optimum mix changed as the level of investment moved from a lower to an upper bound on investment.



13. Having run the LP, the different mixes and deployments were fed back into the PARADE simulation to check that the required level of deployment consistency had

been achieved, and to generate measures of AD protection achieved. The results showed that the LP achieved consistently effective deployments that provided a more balanced defence against varying combinations of threat tracks. The capability of the LP to forecast the effectiveness of the different mixes was so good that, for all but a small number of spot checks, the LP was able to be used as the primary analysis tool. This saved a great deal of time and effort in running the simulation model.

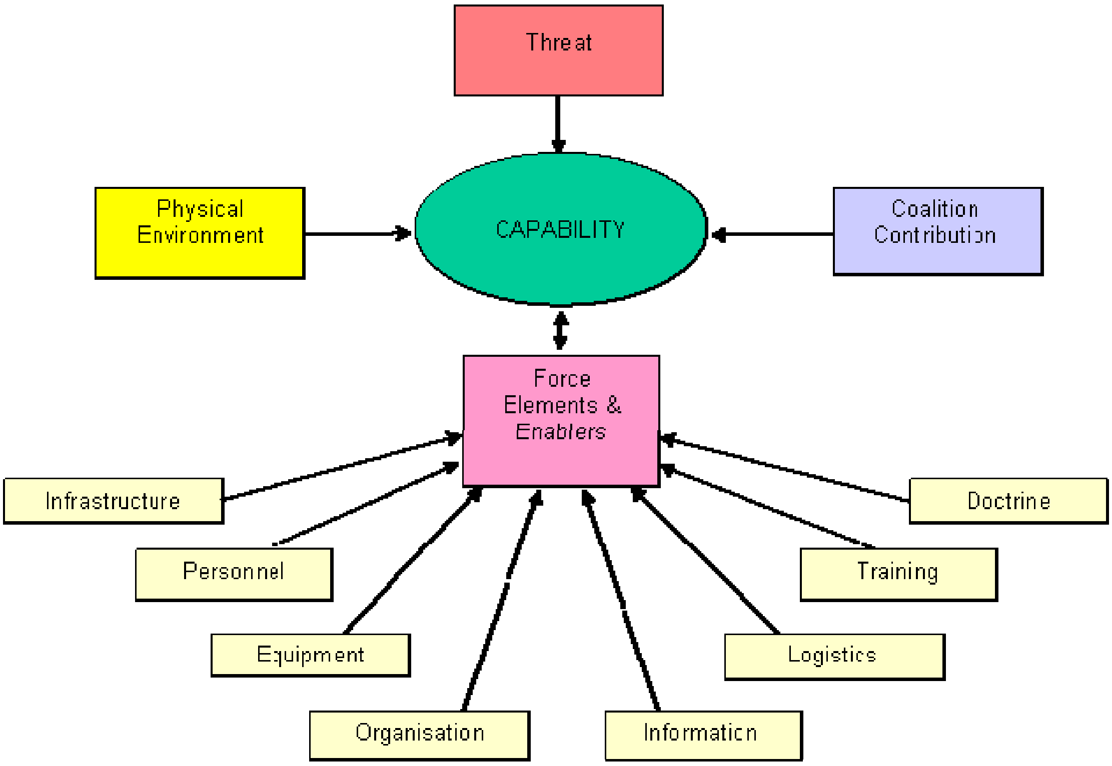
14. The LP model also showed up imbalances in the selection of potential AD sites. Tracks that had the fewest potential kills against them were identified and further sites that could engage them were identified. While these were often poorer in terms of area coverage, they enabled particular avenues of approach to be defended against and overall performance to be improved.

<u>Strategic BoI Study (2007-9)</u>

15. The most important question that the MoD currently faces is:

> "What is the most cost-effective mix of Force Elements at Readiness and Force Enablers that will enable the UK to meet the range of operations required by current Defence Policy?"

This requires firstly that we understand the relationship between investments in such assets as manpower, training, equipment, facilities, information systems, and logistics to the delivery of military capability in operations. As this figure shows, the capability delivered is dependent not only on that investment, but on the threat, the operational environment and the contribution of coalition partners.

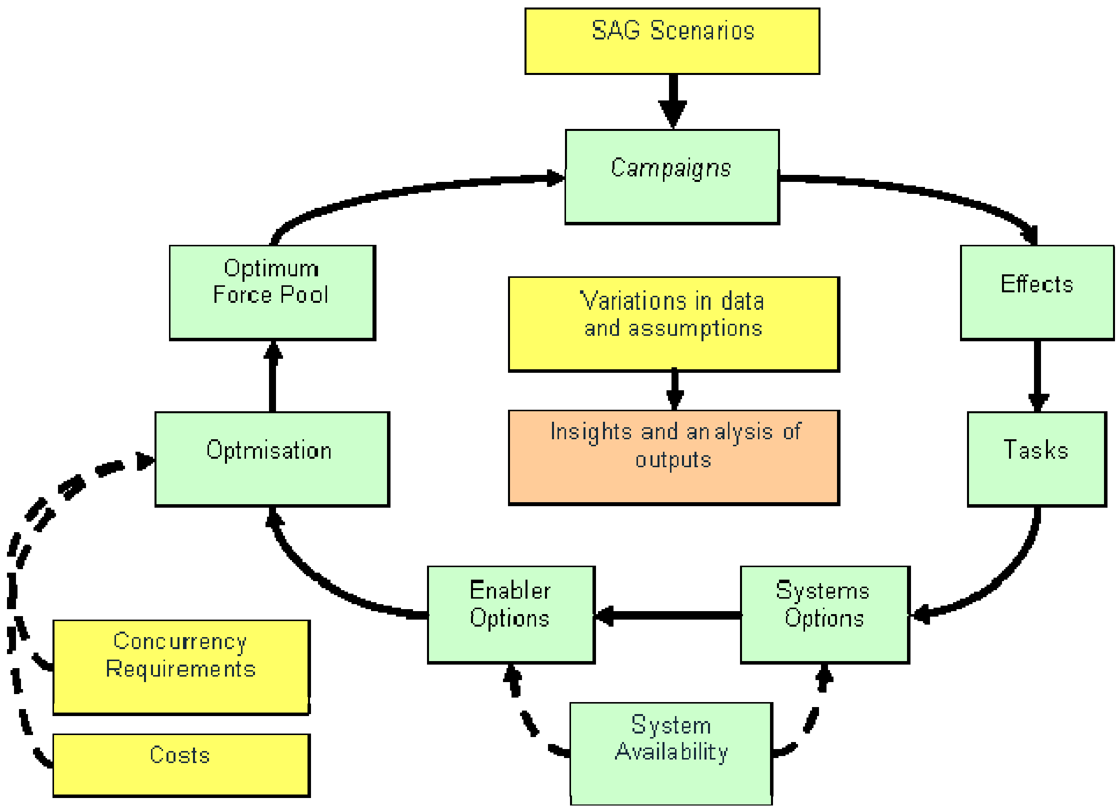


16. To achieve this we have identified an endorsed set of representative planning scenarios against which delivered capability can be tested in a range of operational contexts and have developed corresponding campaigns in high level simulation models and wargames. Secondly, a method is required to explore the solution space of possible combinations of force elements and force enablers that might achieve

campaign success in the full range of potential future sets of concurrent operations. Force elements include maritime and air platforms and land force units at company/squadron level. Force enablers include strategic transport, logistic support, ISTAR and C2.

17. In the past we have tried to achieve this by using military judgement to provide a Force Estimate (FE) of the size and composition of the force package required to conduct a given operation. A spreadsheet based tool was then used to identify the extent to which planned forces could meet all of the possible sets of concurrent operations allowed by Policy. Possible adjustments to both the requirements and the plan were then identified by manual inspection that would reduce shortfalls and improve efficiency. This had the disadvantages of being both highly subjective and of only exploring marginal changes.

18. Following on from three successful pilot studies that optimised investment in selected capability areas or against a limited mission set, we are now in the process of conducting a Strategic Balance of Investment Study. This uses a linear programming approach to answer the second of my questions above. The study process is illustrated in this diagram.



19. The complete set of about 40 endorsed planning scenarios were reviewed to identify a manageable subset that would be representative of the range of operation types, physical environments, threats and coalition types that might be expected to arise within current planning assumptions. For this subset, successful campaigns, informed by endorsed Operational Concepts, were generated using high level campaign simulations and wargames. These campaigns were then decomposed to identify the required effects and the actions that achieved those effects, for each phase

of the campaign. Each action was characterised as a task requiring appropriate force elements and force enablers to accomplish it.

20. In parallel, a list of the potential current and future force elements and enablers in which the UK might invest was compiled. This was then mapped onto the campaign task lists to establish which force elements would be capable of undertaking each task. The associated requirement for force enablers was also assessed. These may be discrete requirements such as ISTAR to support a particular task, or general requirements such as strategic transport or C2 associated with the deployment of a given ground manoeuvre force to a theatre. In order to address the evolution of forces over time, a number of epochs may be defined, typically about 5 years in length. In and out of service dates for force elements are defined and the mapping of force elements to tasks is repeated for each epoch.

21. Policy requires the capability to undertake intervention or power projection operations whilst maintaining enduring peace keeping or stabilisation operations and standing tasks. Various combinations of scale and type of such concurrent operations are defined. These may vary from epoch to epoch. For each force element and force enabler the fixed and variable components of its whole life cost are identified.

22. These data provide the framework on which a linear programme has been constructed that will identify the least cost mix of force elements and force enablers that is capable of undertaking all of the standing tasks and sets of concurrent operations called for by policy. Such a tool can be used rapidly to explore the solution space, identifying the force driving tasks and campaigns, and testing the sensitivity of the solution to uncertainties in cost, in-service dates, system performance and campaign requirements.

Conclusions

23. The following benefits can be obtained by using optimising models, and in particular Linear Programmes, in military Operational Research studies:

a. The combination of simulation and LP offers a very powerful approach, allowing the whole of the solution space to be explored and those options that offer the greatest benefit to be the subject of detailed examination using simulation models or wargames.

b. Options can be compared on a consistent basis, with assets employed equally well, removing an element of player variance.

c. LP models can be rapidly formulated and implemented using standard software packages

d. RHS parametrics allow rapid exploration of the solution space, allowing extensive sensitivity testing to be conducted.

e. A MAXIMIN or MINIMAX objective function provides solutions that are robust to uncertainty.

f. Potentially non-linear problems can be solved by using an LP formulation iteratively.