

Many thanks to ISMOR organizers for arranging this lovely venue and such a worthwhile conference. I am delighted to be here today.

For those I have not already yet met, I am an operational research analyst for the Canadian Army and work in Kingston, Ontario.

This presentation is a summary of material in a paper that Geoff Pond and I co-wrote on the topic. For more details, the paper will be available from the ISMOR archive download site.



## The Origins of Schools of Thought Analysis



- Brigadier-General Mike Jeffery's decision style
- The Canadian Army budget crunch of the 1990s:
  - Fund the maintenance of existing capabilities
  - *And* find resources to invest in the future
- Innovative process started in Kingston, Ontario to incorporate creative thinking and critical thinking
- Decision-analysis methods under consideration:
  - Most focused on mathematical manipulation of individual preferences to generate the group's preferences
- General Jeffery wanted to incorporate input from dissidents, from contrarians, from rebels, from mavericks

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

Schools of Thought Analysis or SOTA originated with some dissatisfaction that Brigadier-General Mike Jeffery had with decision-analysis methods in the 90s.

At the time Gen Jeffery was wrestling with two competing demands for limited resources. The Canadian Army wanted to maintain its existing capabilities. But some visionaries, including Gen Jeffery, needed resources to start investing in the future. Meanwhile there was a looming budget crunch. The mandate he had at the time from the then Army Commander was to look to the future of the Army, which in the 1990s was looking a bit fuzzy.

As it turned out, Gen Jeffery had a continuing interest as he later became Army commander, and confronted these conflicting budget issues at a higher level.

In **1996** Gen Jeffery had just set up a multidisciplinary team in Kingston to look to the army's future and to provide him with advice. The team's composition drew in some of the more cerebral uniformed staff, but also representation from R&D, from academia, from the other services. He even included an OR analyst.

A decision-analysis overview given to General Jeffery covered methods that were largely intended to provide a mathematical basis for combining ranks or scores from individuals into some overall indication of group preference.

General Jeffery was clear that this was *not* what he was seeking at all. He wanted a mechanism that would ensure that lone voices would be heard and not suppressed by some majority. He was keen to learn from all manner of input, regardless of source. If 'pony-tailed hippies' were prepared to offer worthy ideas, General Jeffery was prepared to listen. He *wanted* to hear contrary views.



## Student Evaluation of Lectures



Title of Lecture	Abbreviation	Selected Individual Ranks – by student initials							
		jrjw	rmk	cl	cgk	krr	amm	wrd	bkm
1. Battle of Midway	Mid	2	3	4	1	4	4	3	6
2. Bauman's Inferno	BInf	3	5	1	6	5	3	7	2
3. History of Wargaming	His	7	2	6.5	2	7	6	2	3
4. Interim Brigade Combat Team	IBCT	5	7	3	6	6	1	4	4
5. Irregular Warfare	IW	1	6	2	6	1	2	5	1
6. Second Rise of Wargaming	2Ris	6	1	6.5	3	2	6	6	5
7. Systemic Operational Design	SOD	4	4	5	4	3	6	1	7

Note:

- 'cl' has 'His' and '2Ris' tied in last place
- 'cgk' and 'amm' have three-way ties for last place

SOTA is best explained with an example.

Here we have the results of a small survey of how eight students in an operational research course at the US Naval Postgraduate School ranked seven of their lectures in terms of potential value in their future careers. A value of “1” means “most preferred”: *first place*. Note: there are many ties.

For comparison to how SOTA was used by the Future-Army thinkers in Kingston, the lecture titles might be proposed concepts, and the individual ranks could come from participants in a group brainstorming exercise trying to determine the best investments for the future.

Note that the raw ranks have been replaced by what can be called canonical ranks – the sum of ranks for each participant is  $n(n+1)/2$ , where  $n$  is the number of alternatives.

For ties in Kendall's canonical form, the entry is the average for the positions had they not been tied. That is: if two items were tied for first, they would each get  $1 \frac{1}{2}$  (average of  $(1+2)/2$ ). Or, see that 'cl' has two lectures in last place, these are each given a value of  $6 \frac{1}{2}$  or  $(6 + 7)/2$ . And 'cgk' and 'amm' have both have three of the lectures tied for last place. So those three get a value of  $(5 + 6 + 7)/3$  or 6.



## Group Ranking



Lecture	Rank Sum	Group Rank
IW	24	1
Mid	27	2
BInf	32	3
SOD	34	4
2Ris	35.5	5.5
His	35.5	5.5
IBCT	36	7

- Rank sum (of ranks in canonical form) provide the group ranking: lowest rank sum in first place
- Lectures have been reordered by rank sum
- Note '2Ris' and 'His' in tie for 'fifth'
- Group Rank re-named 'Borda'

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

From the previous table we can sum across the rows to get the “rank sum” for each of the alternatives.

Here we have the seven topics reordered by rank sum.

Later we will give the group ranking, in this case with “IW” in first place and “IBCT” in last place, a name: “Borda”.

This is to honour Jean-Charles de Borda (1733-1799), a French mathematician (and military engineer, and naval captain, and scientist). Borda proposed an analogous method, called the “Borda Count”, for multi-candidate voting in the early days of the French Revolution.

The voting method actually precedes Borda. One earlier proponent was Nicolas of Cusa (1401-1464). During his era, he proposed this method for electing Holy Roman Emperors, but it was rejected by the Roman Catholic Church at the time. Documents discovered in 2001 show that Ramon Llull (1232-1315, aka Raymond Lull) was also aware of this voting method, and also of a competing method called the Condorcet criterion (named for the Marquis de Condorcet, a contemporary, a countryman, and a rival of Borda).



## Statistical Methods



- Kendall's Coefficient of Concordance,  $W$
- Friedman's Test
- Kendall's Rank Correlation Coefficient (with ties),  $\tau_b$
- Distance metric from coefficient:  $d = (1 - \tau_b)/2$
- Hierarchical cluster analysis:
  - simple linkage or nearest neighbour
  - complete linkage or furthest neighbour
  - average
- Multidimensional Scaling (MDS)
- Coded as Excel macro and R commands

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

Now that we have some *numbers*, where can we go with them? First we can go back to Sir Maurice Kendall for his coefficient of concordance,  $W$ , and a test of its statistical significance. This will tell us whether the eight students show enough consistency that we can conclude their rankings are not merely a random selection. A similar test was developed in parallel by Milton Friedman.

The rankings of the eight students results in a  $W$  of 0.07. Note  $W$  will range from 0 to 1, with 1 indicating complete agreement amongst all judges. A value of  $W$  as small as this would already be an indicator of very little consensus. Testing the significance, we find we cannot reject  $H_0$ . That is: we must admit that that the eight rankings could be from no more than random ordering.

When reporting back the group ranking, we would certainly need to highlight such results of this statistical test. Here, one should be very careful about assuming that a similar group of students would produce a similar result for a group ranking. However, let us use this example to explore the data further.

In this we will use pairwise rank correlation coefficients, namely, Kendall's  $\tau_b$ . From these coefficients, we derive distances and go on to use these in three forms of cluster analysis and in multidimensional scaling. Note: if the coefficient is -1 it maps to a distance of 1, and a coefficient of +1 maps to a distance of 0.

The results of using the data for cluster analysis and MDS are best illustrated diagrammatically as we shall see.

For our applications, we have coded much of the analysis into an Excel macro, and you will see that the statistical language R provides convenient ways to do cluster analysis and MDS.



## Pairwise Rank Correlation Coefficients



	jrj	cl	krr	amm	bkm	cgk	wrd	rmk
cl	<b>0.59</b>							
krr	<b>0.43</b>	0.00						
amm	<b>0.41</b>	<b>0.63</b>	-0.10					
bkm	0.14	<b>0.39</b>	-0.05	<b>0.41</b>				
cgk	<b>-0.31</b>	<b>-0.58</b>	-0.10	<b>-0.50</b>	<b>-0.41</b>			
wrd	-0.14	<b>-0.49</b>	-0.14	<b>-0.31</b>	<b>-0.43</b>	<b>0.31</b>		
rmk	<b>-0.33</b>	<b>-0.59</b>	0.05	<b>-0.82</b>	<i>-0.24</i>	<b>0.62</b>	0.05	
Borda	<b>0.78</b>	<b>0.40</b>	<b>0.39</b>	<i>0.21</i>	0.20	-0.05	-0.10	-0.20

### Note:

- Coefficients range from -1 to +1, with +1 indicating two rankings are the same, and -1 that one ranking is the exact reverse of the other
- 'jrj' agrees strongly with 'Borda'
- 'amm' disagrees strongly with 'rmk'

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

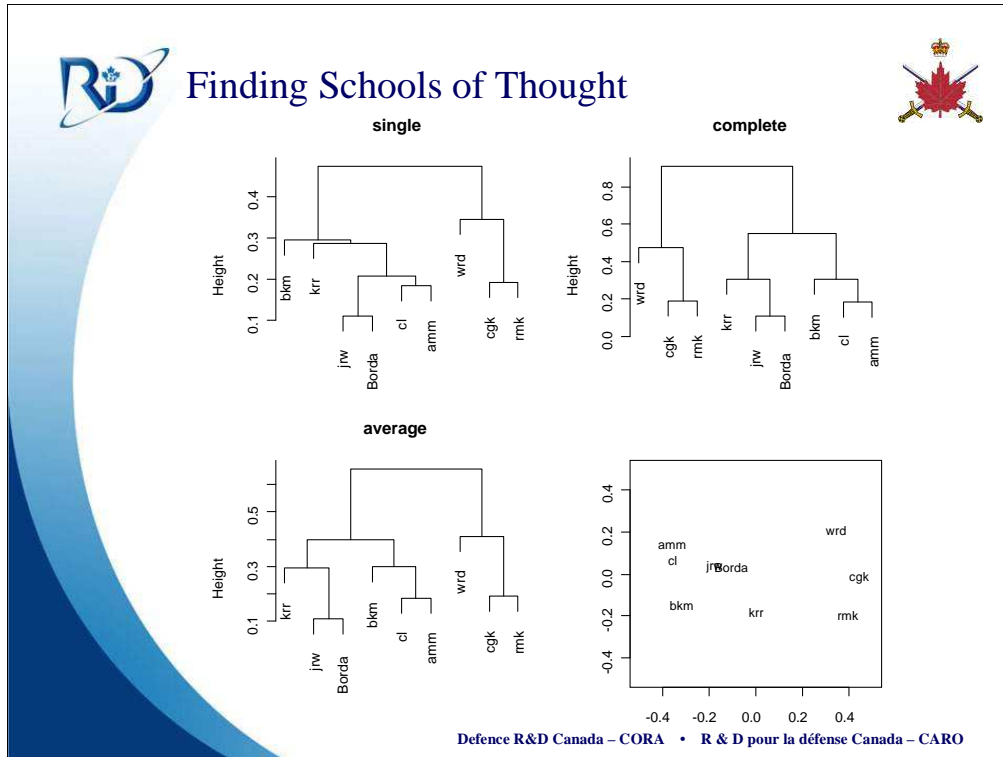
Here we have a table of all pairwise rank correlation coefficients from the participants, with “Borda” added to represent the ranking on the previous slide – the group ranking using rank sum. Values coloured red and in bold are statistically significant at the 1% level, and those in orange and italics are significant at the 5% level. Those in black are not statistically significant; we may use the term “ambivalence” to describe the weakness demonstrated in such a relationship.

So, we now have a means of determining, in a rigorous way, how strongly individual pairs agree and also of how strongly each individual agrees with the group ranking (the bottom row). BTW, the Excel macro reorders the participants by their agreement with Borda, hence the colour change across the bottom will be from positive red, through black, to negative red (when present).

We may note that four of the eight students have some considerable agreement with the group ranking (“Borda”), and the remaining four are ambivalent.

We might also note that most of the coefficients across the rows for ‘cgk’, ‘wrd’, and ‘rmk’ are negative, except at the end (for three pairs involving two of the these three students – the values ‘0.31’ (red)), ‘0.62’ (red), and ‘0.05’ (black).





Here we have the diagrammatic results. We use three variants of cluster analysis. There are many versions of cluster analysis provided in the statistics literature. They each have strengths and weaknesses in illustrating the underlying structure in data.

Two of the methods we use are called “single linkage” or “nearest neighbour”, and “complete linkage” or “furthest neighbour”. Since they use almost opposite criteria for combining two sub-clusters into one new sub-cluster, they provide perspectives that are at opposite ends of a spectrum. The average method is something of a compromise between these two.

Multidimensional scaling (or MDS) is a means of reducing the dimensionality of data. With seven lectures to rank, we might view each of the students having a location in 7-space, with each axis labelled with the title of a corresponding lecture. The coordinate for any individual in this 7-space could be determined by the value of the corresponding lecture. If two individuals had similar values for the lectures the coordinates in 7-space would be similar and they would be in close proximity. Conversely, if they had substantial differences in valuing the lectures, the coordinates on any one axis would be quite different, and the location of the two individuals in 7-space would be at some considerable distance from each other.

The idea of MDS is take the configuration from  $n$ -space and redraw it in something like 2-space, where we humans can better interpret the configuration. Here, for example, we see that ‘jrw’ is close to ‘Borda’ and ‘rmk’ is quite a distance away, as are ‘wrd’ and ‘cgk’.





## The Math Behind the Methods



- Cluster analysis

- Join two sub-clusters  $C_i$  and  $C_j$  where distance (height) between them is a minimum for all pairs of sub-clusters:

- Single Linkage or

- Nearest Neighbour:  $\min(d_{ij}) \quad \forall i \in C_i, j \in C_j$

- Complete Linkage or
  - Furthest Neighbour:

- $\max(d_{ij}) \quad \forall i \in C_i, j \in C_j$

- Average Linkage:

- $\frac{\sum d_{ij}}{|C_i| \cdot |C_j|} \quad \forall i \in C_i, j \in C_j$

- Multidimensional Scaling

- Minimize Stress ( $S$ ), where:

- $$S^2 = \frac{\sum_{i < j} (d_{ij} - \hat{d}_{ij})^2}{\sum_{i < j} d_{ij}^2} \quad \text{for } i, j = 1, \dots, m$$

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

This is a snippet of the mathematics behind the cluster analysis and multidimensional scaling used in SOTA. The three cluster analysis methods, in order, are widely attributed to PHA Sneath, T Sorensen, and RR Sokal and CD Michener. This form of MDS is due to JB Kruscal.

One can see that single linkage and complete linkage are substantially different. In single linkage, if an item in one sub-cluster and an item in the other sub-cluster have a particularly small distance between them, which then becomes the distance between the sub-clusters, those two sub-clusters will be combined. Note that other pairs may be at a considerable distance apart, but will make no difference to the combining.

In a sense, the reverse will happen with complete linkage. If there is one pair on opposite sides of the “boundary” between the candidate sub-clusters which happen to be a considerable distance apart, that distance will set the distance for the two sub-clusters. Thus two such sub-clusters are unlikely to come together until the end, all because of the “repulsion” between those two particular items.

The “average” method is exactly that: a sort of compromise between the single and complete methods, using an average of the distances.

MDS starts with a configuration of the items typically in 2-space. The idea is that to have distances in 2-space be as close as possible to the distances in the original  $n$ -space. Unless the case is trivial, there will be some distortion as points are moved around in 2-space to find a good solution. The stress function is a measure of that distortion. MDS usually works with an optimization algorithm that tries to improve the configuration in 2-space until stress is a minimum.



Issue	Participant Identifier									
	Maj-C	Col	Maj-R	Maj-S	Civ-S	Civ-T	Maj-F	MWO	Civ-R	Borda
Soldier	1	1	2.5	1	1	1.5	2.5	1	4	1
Mobility	2.5	2	1	2.5	3	1.5	2.5	2	5	2
Pwr Mgmt	2.5	3.5	2.5	4.5	4	4	2.5	6	1	3
Sustain	4	3.5	4.5	2.5	2	4	6	5	6	4
C2-CIS	5.5	6	4.5	4.5	5	6	2.5	3	3	5
Equipment	5.5	5	6	6	6	4	5	4	2	6

Note:

- Most rank ‘Soldier’ highly, but not ‘Civ-R’
- Most rank ‘Equipment’ low-ish, but not ‘Civ-R’
- Many submitted multiple ties

Here we have results from a second example. Nine participants in a brainstorming group developed descriptions of candidate investments in R&D for Arctic operations. Without going into details, ‘Soldier’ represents a group of initiatives associated with individual soldiers, e.g., ranging from load carriage techniques, clothing, tentage, to rations tailored for Arctic conditions. Other code words were associated with other potential ‘investment packages’ or issues.

The rank sum has been calculated and used to reorder the alternatives by the Borda method, and the ranks for ‘Borda’ have been added in the last column, from most preferred to least preferred.

Note that, by inspection, we might already expect ‘Civ.R’ to be something of a contrarian.

Kendall’s coefficient of concordance,  $W$ , in this case is 0.42 which is statistically significant well beyond 0.01. In fact, in an  $F$ -test the  $p$ -value is 0.0004. Hence this time we can reject  $H_0$  and conclude that there *is* considerable concord amongst the participants.

As mentioned  $W$  will range from 0 to 1. When the number of judges or participants is large, say more than 10, values of  $W$  that are substantially less than 0.5 may still test as significant. So merely looking at where  $W$  is on the 0-1 interval may be misleading as an indicator of agreement.



## Pairwise Rank Correlation Coefficients



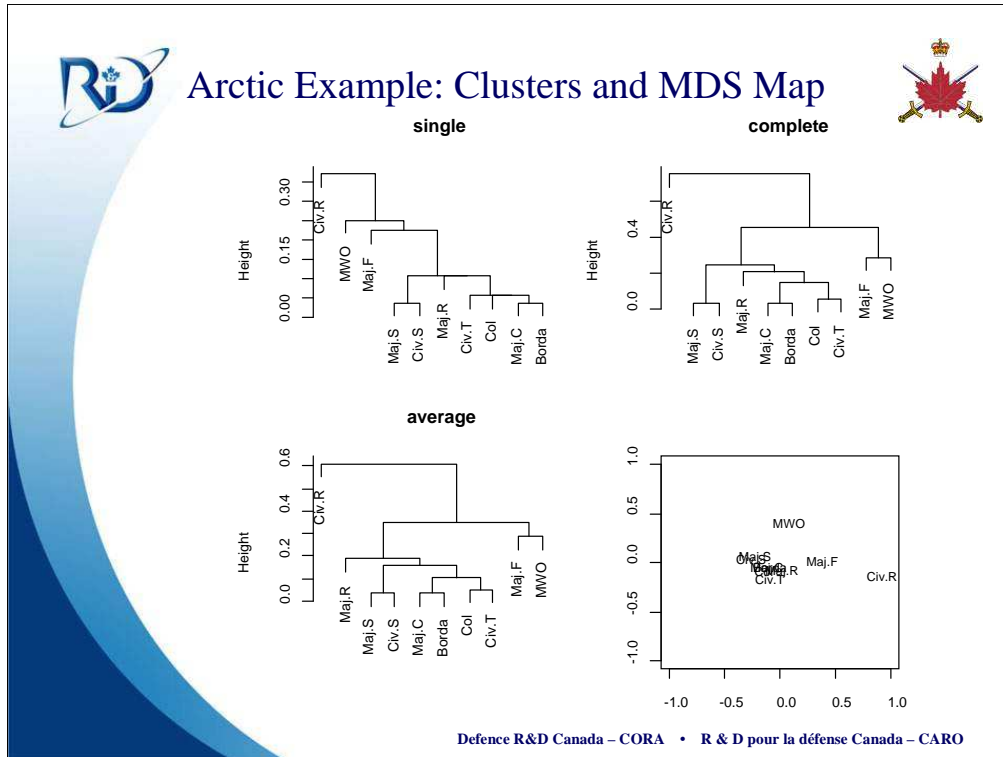
	Maj-C	Col	Maj-R	Maj-S	Civ-S	Civ-T	Maj-F	MWO	Civ-R
Col	<b>0.89</b>								
Maj-R	<b>0.69</b>	<b>0.59</b>							
Maj-S	<b>0.69</b>	<b>0.74</b>	<b>0.54</b>						
Civ-S	<b>0.64</b>	<b>0.69</b>	<b>0.50</b>	<b>0.93</b>					
Civ-T	<b>0.75</b>	<b>0.89</b>	<b>0.59</b>	<b>0.59</b>	<b>0.54</b>				
Maj-F	<b>0.37</b>	0.18	<b>0.55</b>	0.18	0.09	0.20			
MWO	0.21	<i>0.28</i>	0.21	<b>0.50</b>	<b>0.33</b>	<b>0.39</b>	<b>0.43</b>		
Civ-R	-0.07	-0.14	-0.21	<b>-0.50</b>	<b>-0.47</b>	-0.23	<i>0.26</i>	<b>-0.33</b>	
Borda	<b>0.93</b>	<b>0.83</b>	<b>0.79</b>	<b>0.79</b>	<b>0.73</b>	<b>0.70</b>	<b>0.43</b>	<b>0.33</b>	-0.20

Note:

- Considerable agreement with 'Borda' – statistically significant for all except 'Civ-R'
- 'Civ-R' has mostly disagreement with other individuals

Here we have the table of Kendall rank correlation coefficients. Certainly, of the nine participants, 'Civ-R' has the least agreement with the group ranking (see 'Borda' across the last row).

However the remaining eight seem to have considerable internal agreement: the coefficients in the first seven rows are positive and most test that there is statistically significant pairwise agreement (positive red values).



These are the SOTA diagrams for the nine participants in this activity.

We can see six of the nine generally constitute a sub-cluster of considerable agreement. In the MDS map in the lower right. They are in such agreement that the labels for the six are highly concentrated.

As would be expected “Civ.R” can be viewed as an outlier – last to join the final cluster in all three dendrograms, and at some distance from the majority in the MDS map.

In discussion amongst the group we may wish to draw out ‘Civ.R’ for his contrarian views. We might also want to engage ‘Maj.F’ and ‘MWO’ for their insights – they agreed substantially with the majority of six, but ‘from a distance’.



## Giving Voice to Dissent



- With diagrams map of agreement and disagreement, invite contrarians to speak
- Some contrarians, seeing allies nearby, may be more willing to speak: no longer ‘crying in the wilderness’
- In many professional groups, with mutual credibility and respect already established, contrarians have generally been willing to speak up when invited
- Views from contrarians may indicate unanticipated weaknesses in the preferred alternatives
- Iteration of SOTA may indicate convergence towards some worthwhile compromise
- SOTA has some relevance to ‘value-focused thinking’: if two individuals are contrary in their ranking of objects, what does that say about their values?

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

SOTA can contribute to the further debate on the alternatives, and even to a reconsideration of the criterion used to evaluate them.

The methods help identify where the contrarian opinions lie. Some idea of sources of disagreement might come from simple observation of the table of individual ranks. But, while the table looks simple, it represents multivariate data – where the number of dimensions equals the number of items. We are working in  $n$ -space, where  $n$  may be a relatively large number.

During the quest for someone to speak with a contrarian voice, it often helps the contrarian to see that he or she has potential allies, and is not a ‘voice in the wilderness’. The potential allies may not have identical views, but they may be closer to a contrarian than some majority of like-minded participants.

While some methods, like Delphi, often rely on anonymity to encourage individuals to offer their thoughts, SOTA does rely on each individual seeing where they are relative to the others. In most of our applications, the participants often have mutual credibility with others in the group, and considerable self-confidence to speak up on their views.

SOTA allows the identification of participants who may be well placed to apply *critical thinking* to the apparently preferred options – always a valuable contribution in a planning phase.

Of course, if some participants were to use assertiveness, rank, or eloquence to suppress the views of the contrarians, some caution would be required. Nevertheless SOTA would provide indications of contrarian thinking, even if some aspects of the group dynamics might cause potential critical thinkers to remain silent.



## Lessons from a Decade of Experience



1. Acknowledge there *will be* dissenting opinions and contrary points of view
2. Apply a method to determine level of consensus or concordance (or lack of it)
3. Use a non-threatening sample problem to engage the participants and to allay misgivings about process
4. *Making* a decision is not the objective, implementation *is!*
5. Advocate critical thinking during the implementation phase (as well as during planning)

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

SOTA anticipates there will be dissenting opinions and contrary points of view. In most deliberative groups there always is – so get over it!

SOTA includes a means to determine if a sufficient consensus has been reached (for ranks, this is Kendall's coefficient of concordance,  $W$ ). If there is not much concord, this fact should be shared so no one is under the illusion that the group rankings have the universal and emphatic support of all participants. If the concord is weak, it may be time for more investigation and not for some abrupt and poorly supported decision just because the math will spit out a result.

In a previous application, we found benefit of applying the method to a non-contentious sample problem. On one occasion we used SOTA at the Officers' Mess in blind taste testing of local wine. SOTA gained considerable credibility from this that carried over to the real problem the next day.

Sometimes, our profession needs to exercise a bit of modesty: not everyone will have confidence in a decision-analysis method merely because it has credentials within the OR community. Non-analysts need to see a method applied to something they *really* understand, say, buying a new car, or tasting wine.

We should also have some modesty in appreciating that making the decision is *not* the ultimate objective. Implementation is! So if group dynamics have been ruined during planning, is it little wonder that those who were dissenters during planning become subversives during implementation?

That said, we *can* find useful critical thinkers through an application of SOTA. These may later be employed as "canaries in the mine shaft". "Sir, the plan is no longer working as we had hoped. Should we try something different?"



## Conclusions and Recommendations



- The stimulus:
  - A general officer was seeking more than decision analysis mathematical techniques generally provide: Can we identify and incorporate contrary points of view?
- Practitioners should apply traditional decision-analysis methods
- But they should also apply diagnostics to determine:
  - Possible lack of consensus
  - Sources of dissenting opinions
  - Reasons for alternate points of view
- Because they add value to the process, give voice to:
  - Dissidents
  - Contrarians
  - Rebels
  - Mavericks
- SOTA provides a suitably rigorous method

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

SOTA started with a general officer and his concerns that decision-analysis methods were too focused on mathematical procedures to take individual preferences and provide group preferences. He wanted to ensure that he would hear of contrary points of view.

Traditional decision-analysis certainly still has a role within the profession of operational research. Indeed most deliberative groups will apply some home-grown method if there is nothing else provided.

We must all have seen the approach of agreeing a criterion (perhaps “min cost”) and having each judge score the alternatives on, say, a scale of 1 to 10. Then the “winning alternative” is the one with the best total score from a roll-up of scores from all of the participants.

(Applied blindly such approaches often result in the best compromise winning “the competition”. Remember: “a camel is a horse designed by a committee”.)

Of course, there are many others techniques for combining individual preferences into one group preference. The OR journals can be consulted for some of the leading examples.

SOTA takes the path of applying diagnostics to the results of such deliberations. We at least want to know if there is a lack of consensus: “Yes, the winner has the highest total score, but, we all agree that no one in his right mind would actually want to choose *that* alternative.”

In particular SOTA provides a rigorous way to identify sources of dissention. And it gives voice to those who have opinions the majority might not anticipate.





## R Source Code: *Do Try This at Home!*



```
> fl <- file.choose() # Choose file produced by Excel
> rr <- read.table(fl,header=TRUE) # Read table in .txt format
> dd <- as.dist((1 - cor(rr,method="kendall"))/2)
> # Calculate Kendall's tau
> # and convert to distances
> cs = hclust(dd,method="single") # Cluster analysis - single
> cc = hclust(dd,method="complete") # Cluster analysis - complete
> ca = hclust(dd,method="average") # Cluster analysis - average
> isomdsloc = isoMDS(dd,maxit=100, # Multidimensional scaling (MDS)
+ tol=1e-10)
> x = isomdsloc$points[,1] # Extract x coordinates
> y = isomdsloc$points[,2] # Extract y coordinates
> windows() # Open graphics window
> layout(matrix(c(1,2,3,4), 2, 2, # Four plots in one window, 2 by 2
+ byrow=TRUE))
> plot(cs, main="single",sub="",xlab="")
> plot(cc, main="complete",sub="",xlab="")
> plot(ca, main="average",sub="",xlab="")
> plot(x,y,type="n",xlab="",xlim=c(-0.5,0.5), # Draw frame for MDS configuration
+ ylab="",ylim=c(-0.5,0.5))
> text(x,y,rownames(isomdsloc$points),cex=0.8)
> # Draw identifiers for MDS
```

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

For those who might want to experiment on their own, here is R source code to reproduce the quad-charts in this presentation of cluster analysis and MDS.

The first two command lines select a file and read in the data. The following PowerPoint slide will show you the sample data and the format that is needed for this file.

The third line calculates the pairwise Kendall rank correlation coefficients and transforms them into distances for use in cluster analysis and MDS.

This analysis is then performed one corresponding line at a time.

The remaining lines generate the corresponding graphics, extracting the coordinates for MDS, partitioning a window into four parts, and plotting the three cluster configurations and the MDS map.

For performance, note that, on any mid-power personal computer, the lines of code are processed as quickly as one can copy and paste them into the R command-line processor. (That is, it takes only a few minutes to produce the resulting quad-chart.) It might another few minutes to cut-and-paste the diagram to PowerPoint for sharing with the participants.



## Format of .txt File for R



- Format for the Arctic example:
  - Row 1: Labels, separated by one or more spaces
  - Rows 2 through  $n+1$ :
    - Each row has individuals' ranks with the first object's rank on row 2, to  $n$ th object's rank on row  $n+1$

Civ.T	Maj.S	Col	Maj.C	Civ.S	Maj.R	MWO	Maj.F	Civ.R	Borda
1	1	2.5	3	2	1.5	2.5	4.5	3	1
4	3	1	1	1	3	2.5	1	6	2
2	2	4.5	5	6	5	1	3	1	3
4	4	4.5	2	3.5	1.5	6	6	4	4
4	6	2.5	4	3.5	6	4.5	4.5	5	5
6	5	6	6	5	4	4.5	2	2	6

Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

The format for the data for the R procedure is shown here for the Arctic example.

The first row has the labels, separated by blank spaces. The subsequent rows have the ranks for each object in turn.

Note that which object is which is not relevant for the processing, so the rows are not labelled by which object they apply to. And, obviously, these six rows could be put in in any sequence.



## References



- Borg, I. and Groenen, P. (1997) *Modern Multidimensional Scaling*. New York: Springer.
- Cameron, F. (1998) *Contrary Schools of Thought within Military Decision-Making Groups*. Shrivenham, UK: International Symposium on Military Operational Research  
<http://ismor.cds.cranfield.ac.uk/ismor1998.htm>
- Conover, W. (1998) *Practical Nonparametric Statistics*. New York: Wiley. Third Edition.
- Craig, S. (2007) 'Reflections from a Red Team Leader' in *Military Review*, March-April. TRADOC Combined Arms Center, Ft Leavenworth, Kansas. 57-60
- Daft, R.L., and Marcic, D. (2009) *Understanding Management*. Mason, Ohio: South-Western Cengage Learning, Sixth Edition.
- Dalkey, N.C., (1969) *The Delphi Method: An Experimental Study of Group Opinion*. Santa Monica: RAND RM-5888-PR
- Everitt, B.S., and Hothorn, T. (2010) *A Handbook of Statistical Analyses Using R*, London, UK: Chapman Hall/CRC, Second Edition.
- Janis, I.L. (1972) *Victims of Groupthink*. Boston: Houghton Mifflin Company
- Janus, I.L. (1989) *Crucial Decisions: Leadership in Policymaking and Crisis Management*, New York: Free Press
- Kendall, M.G. (1975) *Rank Correlation Methods*, New York: Oxford University Press, Third Edition.
- Mason, D.W. (1995). *Application of the Consensus Decision Support (CDSP) Methodology in Prioritizing the 1995/96 Major Development Program*, DLOR-RN-9502, Department of National Defence, Ottawa.
- Pond, G., and Cameron, F (2010). 'The mathematics of school of thought analysis', in progress.
- Ward (2007) 'A new twist on the old talking stick' at <http://business.queensu.ca/centres/qedc/> accessed 7 July 2010



Defence R&D Canada – CORA • R & D pour la défense Canada – CARO

