Modelling and Analysis of Defense Lines of Developments Using Fuzzy Causal Maps with a Practical Example

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Abstract

This paper presents a new model of Defence Lines of Development (DLoDs) and their relationships with Military Capability. The model employs Rule Based Fuzzy Causal Maps (RBFCM) to advise on improvements to resource allocation in order to produce more cost effective and militarily capable Force Elements and Capability Packages. The model operates using complex fuzzy causal relations that cannot be specified using precise mathematical concepts, but can only be described by uncertain, imprecise or incomplete information and knowledge. A high level decision support system which encompasses the RBFCM-based model is implemented. It can be used as a tool to analyse how the main attributes of the Force Element will be impacted by changes in resources allocated to components using the DLoDs framework. Relationships between Military Capability, DLoDs and their key contributing factors are built upon knowledge acquired from defence experts. Advantages of using RBFCM to model military capability, using the Joint Strike Fighter (JSF) UK base location problem as an example, and further implementation of the system are discussed.

1. Introduction

1.1. Literature review

The term Cognitive Map (CM) was introduced in 1976 (Axelrod, 1976). The author proposed a set of solutions to model complex systems with interrelated entities in a form of a cause-effect map. One of the main drawbacks of the CMs proposed was the fact that they relied on quantitative data, which made models very complex and in some cases impossible to generate.

To overcome this problem Kosko proposed an alternative approach – a Fuzzy Causal Map (FCM) (Kosko, 1986). In contrast to a traditional CM, a FCM handles qualitative data. The FCM is represented in a form of a directed graph with concepts (nodes) having values ranging from [0, 1] or [-1, 1]. There are two types of nodes: a causal node that has a causal influence on another node and an effect node that is subject to that influence. Links between the nodes have associated weights ranging from [-1, 1], representing directions and strengths of the causal relationships between the nodes. A positive weight represents a causal increase and a negative weight represents a causal decrease. To calculate the value of the effect node, the value of each causal node has to be multiplied by the weight of the link between the two nodes. This makes the FCM's relations between nodes, linear and symmetric.

FCMs have been widely used in a diverse range of applications, such as:

- Engineering (Stylios and Groumpos, 1999),
- Strategic planning (Andreou et al, 2005)
- Information Technology (Rodriguez-Repiso et al, 2007),
- Decision-making, project management, and investment analysis (Lee and Choi, 2004),
- Medicine (Innocent and John, 2004),
- Environment/ecology/terrain domain (Liu, 2003)
- Military (Yaman and Polat, 2009), etc.

However, the linearity and symmetrical features of FCMs do not reflect the reality of most relations existing in real-world problems. Rule Based Fuzzy Causal Maps (RBFCMs) is a relatively new methodology which has been introduced in (Carvalho and Tome, 1999). Since then this methodology has been intensively developed. The RBFCMs brought solutions to many problems with which the standard FCMs have not been able to cope. They have been applied to a few different domains, such as:

- Forest Fire Modelling (Carvalho et al, 2006),
- Fishermen's behaviour in a pelagic fishery (Wise et al, 2012),
- Socio-Economic system (Carvalho and Tome, 2009).

One of the main benefits of using RBFCMs is a wide range of types of relations that can be modelled (Carvalho and Tome, 2000):

- Influence Relation – standard IF – THEN relations involving one or more concepts. If there is more than one concept involved, then they have to be combined using standard fuzzy operators AND and OR).
- Fuzzy Causal Relation (FCR) – this relation exists between causal and effect nodes and this is the most typical relation in RBFCMs.

An important issue in FRBCMs is accumulation of impacts. If, for example, concept A and concept B each cause concept C to increase *a little bit*, then concept C will increase *more than a little bit*. If A affects C *a little bit* and B affects C *much*, then C will increase *more than much*. If A and B have approximately equal but opposite effects on C, then C will not change.

Apart from the main two types of relations mentioned above it is possible to model different, more complex ones including (Carvalho and Tome, 2000): probabilistic relations, time-dependent probabilistic relations, possibilistic, possibilistic/probabilistic, similarity and level to change relationships.

One of the important elements of RBFCMs is a time component, which allows the construction of dynamic systems. Every change in a causal node can have an impact on the effect node after certain base time units, where base time is the smallest time period in which it is possible to observe the effects of the changes on the effect node. Depending on the type of the system, the base time can vary from a very short time period – 1 day (in a system that needs very accurate data) to long time periods – 1 year (leads to less accurate analysis) (Carvalho and Tome, 2001).

An important characteristic of RBFCMs is that an input into the map is qualitative data. This is very important as most real-world systems have very complicated structures with very complex relationships among their concepts. Modelling such systems with quantitative data is very hard because, often, it is difficult or even impossible to collect precise quantitative data and it is difficult to describe complex relationships between concepts using precisely specified relations.

When comparing an RBFCM and a FCM, the former is much more complex and computationally demanding. However, this is not an issue nowadays, because the standard computers can easily cope with the required calculations.

An additional advantage of using an RBFCM is its ability to model complex dynamic systems with different meta-states. The whole system can be decomposed into sub-systems, each one representing a meta-state of the system. A sub-model of each sub-system can be a simpler RBFCM. Transition of the system from one to another meta-state takes place when certain conditions are met.

1.2. First Phase of the project

This work was initiated by the UK Ministry of Defence Centre for Defence Enterprise[1] with the aim of designing a new system for the modelling and analysis of the allocation of resources between Defence Lines of Development (DLoDs) and for the analysis of the impact of this allocation upon the military capability of Force Elements (FE) and Capability Packages (CP). The Global Overview of DLOD Interactions and Value Analysis (GODIVA) project was started in May 2012. During the first part of the project the following objectives were achieved:

- Research regarding different methodologies using fuzzy logic, that could be used to model complicated relationships between DLoDs and their Contributing Factors;
- Modelling of the Fuzzy Causal Map representing a Force Element with four DLoDs and nine Contributing Factors;
- Implementation of the reasoning mechanism and a model;
- Testing of the results and final tweaking of the model.

In the first phase of the project a general map (Figure 1) representing a Force Element was created with support of a defence Subject Matter Expert (SME). The map consisted of four DLoDs (Training, Equipment, Personnel and Logistics) and nine Contributing Factors affecting those DLoDs.

[1] The Centre for Defence Enterprise (CDE), part of MOD's Defence Science and Technology Laboratory (Dstl), aims to enhance innovation in defence research and development by engaging the broadest possible audience of science and technology providers.

Between the Contributing Factors and DLoDs two types of relationships existed:

- Simple fuzzy causal relationships between a Contributing Factor and a DLoD,
- Conditional relationship between a Contributing Factor and a DLoD with additional influence of other Contributing Factors (some rules).

IF – THEN rules, that build the causal relationships, require two attributes: one for the causal part and one for the effect part of the rule. The following attributes were used in the map and were used to build the rules:

- Budget – associated with Contributing Factors, allowing an interaction with the user of the software, representing budget allocated,
- (Military) Ability – associated with DLoDs and Force Element, representing ability of DLoD and expressed by a hypothetical value between 0 and 100.

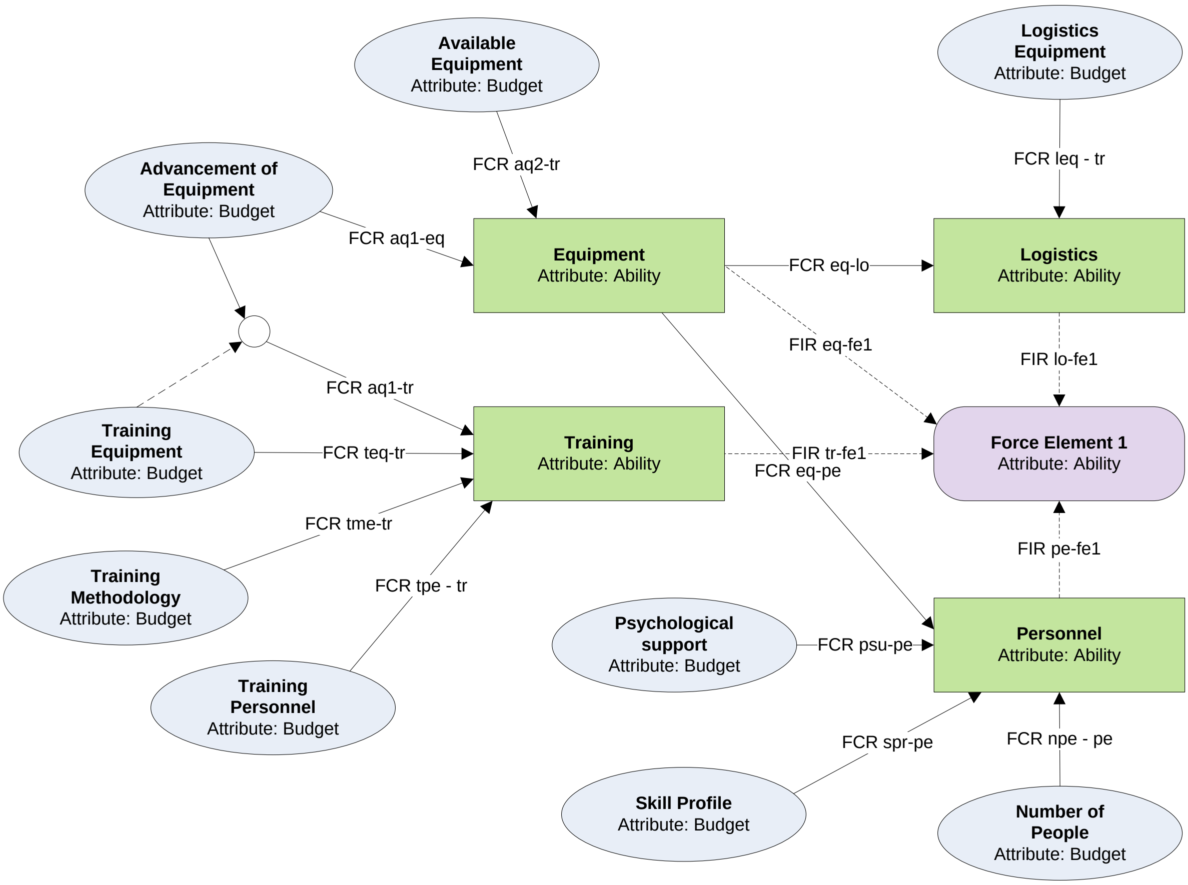

Figure 1. Initial Causal Map

As it can be seen in Figure 1 there are two layers of nodes: Contributing Factors and DLoDs. Users were allowed to interact with the budgets of Contributing Factors only. After modifications of the budget were applied a simulation was run and it was possible to see how changes of the budget affect military ability.

This work was presented at the 29th ISMOR in August 2012.

1.3. Second Phase - Workshops

The first phase of the GODIVA project was considered by Dstl to have demonstrated the potential utility of the RBFCM approach. A second phase was authorised to develop the existing, essentially abstract, model into a model of a specific military Force Element.

At the beginning of the second part of the project, two workshops, with a broader panel of defence experts, were planned in order to further develop the approach. The aim of the first workshop was to familiarize the experts with fuzzy causal map concepts, conduct a brainstorming session to expand the initial model with more realistic contributing factors and to discuss other critical components of the model: membership functions and relationship definitions. During the first workshop it was assessed to be beneficial for the project to focus upon a specific question (rather than on general issues of the military capability generation process). The assessment of options for the location of the UK's future Joint Strike Fighter (JSF) force was suggested. This would require the construction of a feasible model but would not require too much information and therefore be possible to develop in a short period of time.

This model would be designed to enable the analysis of the different resource allocations required for different locations where the JSF force could be based and how these would affect the military capability and/or costs of this force. The first workshop provided a good deal of information about the Contributing Factors, relevant to the location problem, which could affect the JSF force. This was later complemented with further information provided by Dstl via email. It was noted that membership functions (representing budget variations) may differ depending on nodes but at the end of the workshop it was agreed that more important was an expansion of the model to show that it is capable of handling more complex demands.

The second workshop was dedicated to a presentation of the modifications done during the intervening month, verification of the model and methods used and to review the results of the simulation to the panel of experts who took part in the first workshop. This was received very positively and this generated interest in testing the method on a real life problem in a further stage of the project.

2. Development and Implementation of the Model

2.1. Statement of the Problem

At the time of the project, the decision of where to base the UK's future JSF force had been under consideration for some time[2]. For the purposes of this project, the options were considered to be RAF Lossiemouth, in northern Scotland, and RAF Marham, in Norfolk.

For a full cross-DLOD assessment of the two options to be made, a range of factors needed to be taken into consideration. Most obviously, there is the difference in the geographical locations of the two bases. This impacted upon the distances which aircraft would have to fly in order to reach the RAF's training areas – flying fast jet aircraft is an expensive business. There are also other considerations than just flight times, however. Differences in the existing infrastructure at the two bases could have significant impacts upon the developments, construction and associated investments needed to make them suitable to serve as the JSF base.

[2] It has since been announced that the force will be based at RAF Marham (http://www.raf.mod.uk/news/archive/joint-strike-fighter-26032013).

The JSF force is not just composed of equipment and infrastructure. The aircraft also need crews, both in the air, on the ground and in a wide range of supporting roles. These personnel need to be housed and cared for. Many will also have families which will either need to be accommodated on or near the base or the service personnel will need to be compensated for travel costs, boarding schools, etc.

There are other factors to be considered also, so the problem rapidly became significantly more complex than the original abstract map. Dstl worked interactively with the Coventry University team to express and structure the basing problem and build a new Causal Map.

2.2. Causal Map

The RBFCM developed performs an analysis of impact of changes in the budget/number of hours allocated to different DLoDs' components, on the ability of the Force Element under consideration. This requires providing a set of input values and rules as follows:

1. The current budget allocation to DLoDs' components;
2. Changes in the budget/number of hours allocation;
3. Current ability levels of DLoDs and Force Elements;
4. Rules describing how the changes in budgets allocated to the DLoDs' components affect DLoDs; and
5. Importance of ability of each DLoD in determining the ability of the relevant Force Element.

During the second part of the project many improvements had been applied to the model. A fifth DLoD (Infrastructure) has been added to the Causal Map and the number of all Contributing Factors increased to 49. Such an expansion of the map required modifications of the model and the software developed.

The existing causal relationships, that were used to define relationships between nodes, were too simple to capture the complexities of reality. It was not always possible to meaningfully assign budgets to Contributing Factors or to express dependencies between nodes using the causal relationships employed in the initial model. Therefore new types of relationships had to be introduced:

- Linear relationship between (Number of) Hours spent on Training and Budget spent on Consumables and Air-to-air refuelling (AAR), e.g. Squadron Live Flying for Individual Training -> Consumables (Figure 2),
- Linear relationship between (Number of) Hours of different types of training and Overall Live Flying/Synthetic Training (Figure 2),
- Complex causal relationship between two Contributing Factors (the same strength of impact) and a DLoD, e.g. Sqn Live Flying for Individual Training and Sqn Synthetic Training for Individual Training -> Training DLoD (Figure 2),
- Conditional relationship between Contributing Factor and DLoD with additional impact of more than one Contributing Factor.

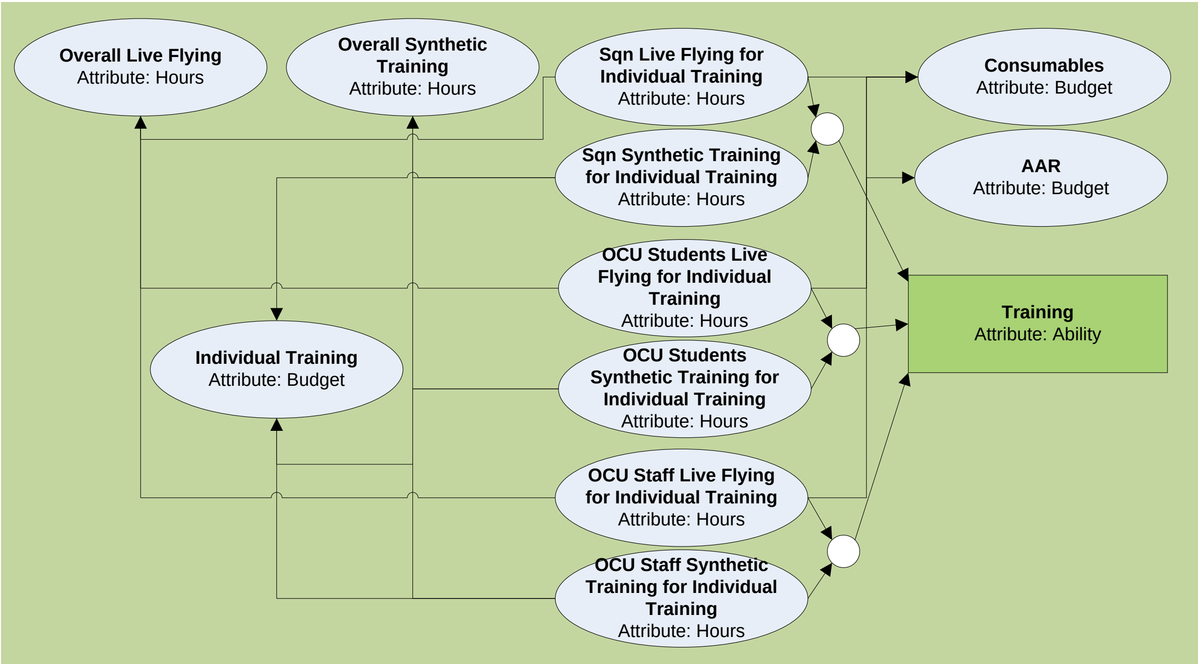

Figure 2. Relationships between training components and Training DLoD/Infrastructure components

Another part of the model that required more information was the representation of some nodes. Using the budget attribute only was not enough to allow the experts to define relationships with the required precision and it was a major limitation brought up by all attendees of the first workshop. Therefore new attributes were introduced to allow a more detailed representation of reality and more detailed simulation and analysis of different allocation of resources:

- Budget – amount of money spent on certain contributing factor,
- Hours – number of hours spent on training,
- Airframes – number of airframes available,
- (Military) Ability – hypothetical value between 0 – 100, expressing how capable a Force Element/DLoD is.

An important improvement in building the map was the introduction of a middle layer (Contributing Factor) between Contributing Factors and DLoD. An example of such a relationship is presented in Figure 2. In the initial map a Contributing Factor could interact only with DLoDs. Change allowing adding more layers was made possible by introducing a new attribute of Airframes.

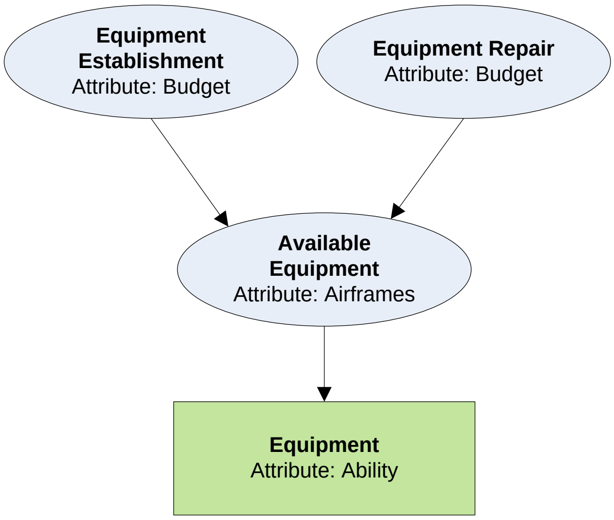

Figure 3. Relation with middle layer between Contributing Factors and DLoD

As mentioned above, improvements greatly contributed to the expansion of the causal map and gave more freedom and flexibility to the military experts to transfer their knowledge into a mathematical model.

The final causal map, developed in the second part of the project can be seen in Figure 4.

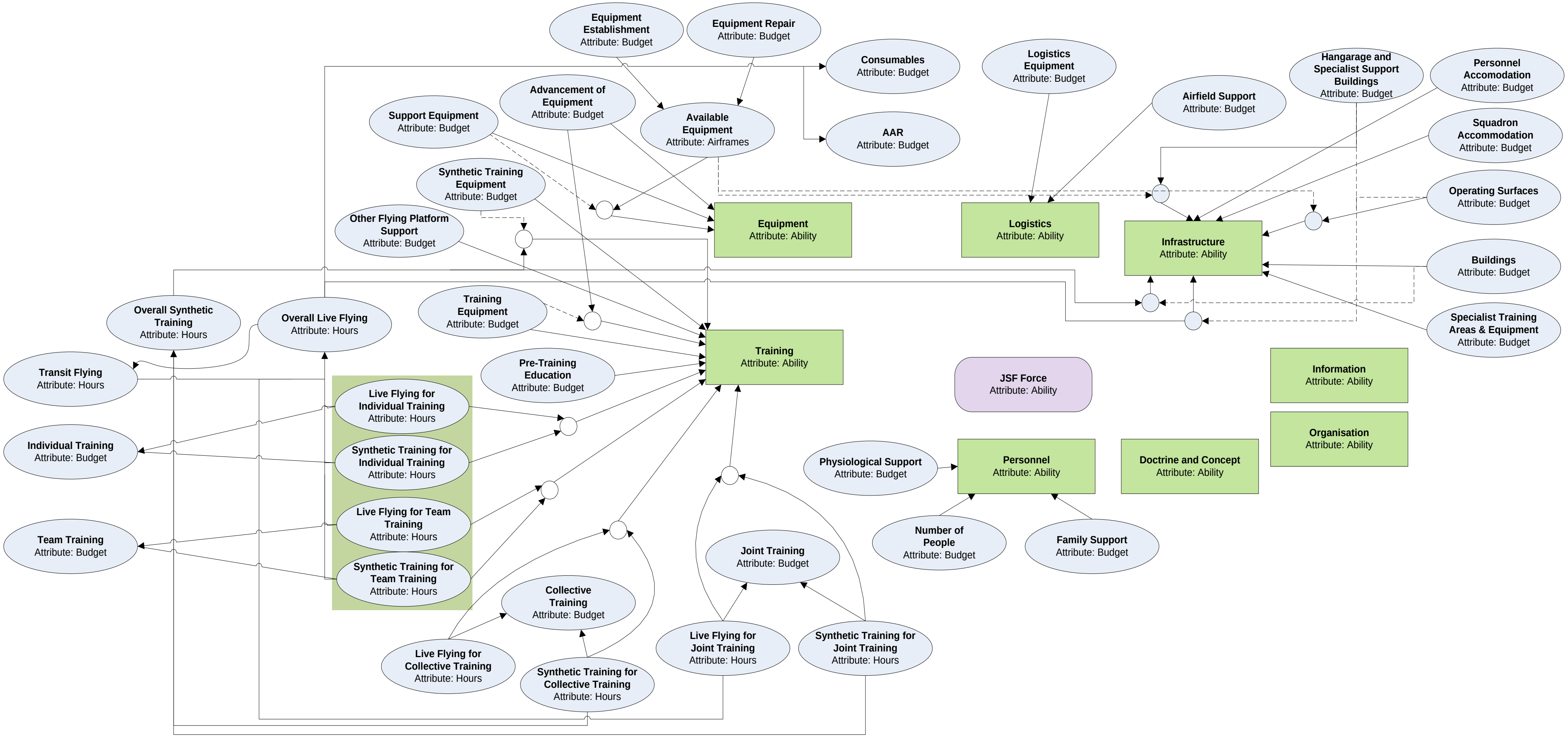

Figure 4. Final Causal Map

2.3. Membership Functions

Fuzzy IF-THEN rules (that build most relationships between nodes) handle qualitative data such as Increase Much (of the Budget), Decreased a Little (of Ability), and so on. The qualitative data are specified using imprecise linguistic terms. Seven linguistic terms were identified for Budget and Ability's variation, including Decreased Much, Decreased, Decreased Little, Maintained, Increased Little, Increased, and Increased Much and five terms defining number of hours: Very Low, Low, Medium, High and Very High. These linguistic terms are formally represented using fuzzy sets (Pedrycz and Gomide, 1998, Yan et al, 1994). For each fuzzy set (defining variation), we defined a hypothetical scale from -100% to +100%, which represents possible percentage of changes either in the allocated budget or ability; -100% represents the maximum possible decrease and +100% the maximum possible increase. The corresponding membership functions, given in Figure 5, represent subjectively determined degrees of belief that a certain percentage of change belongs to a fuzzy set, i.e. the corresponding linguistic term. A degree of belief is expressed as a value from the interval [0, 1]. Naturally, a complete membership of an element to the fuzzy set is represented by the membership degree 1 and a complete non-membership by degree 0. The membership functions are modelled using trapezoidal functions. For example, every change higher or equal to +50% definitely belongs to the fuzzy set Increased Much, but changes between +40% and +50%, belong also to this fuzzy set, but with degrees of belief linearly increasing from 0 to 1, respectively. All changes lower than +40% definitely do not belong to the fuzzy set Increased Much. Fuzzy set Increased is defined in a similar way. All changes between +30% and +40% definitely belong to the fuzzy set Increased; however, changes between +20% and +30% belong to the fuzzy set Increased with degrees of belief linearly increasing from 0 to 1, respectively, while changes between +40% and +50% belong to the fuzzy set Increased with degrees of belief linearly decreasing from 1 to 0.

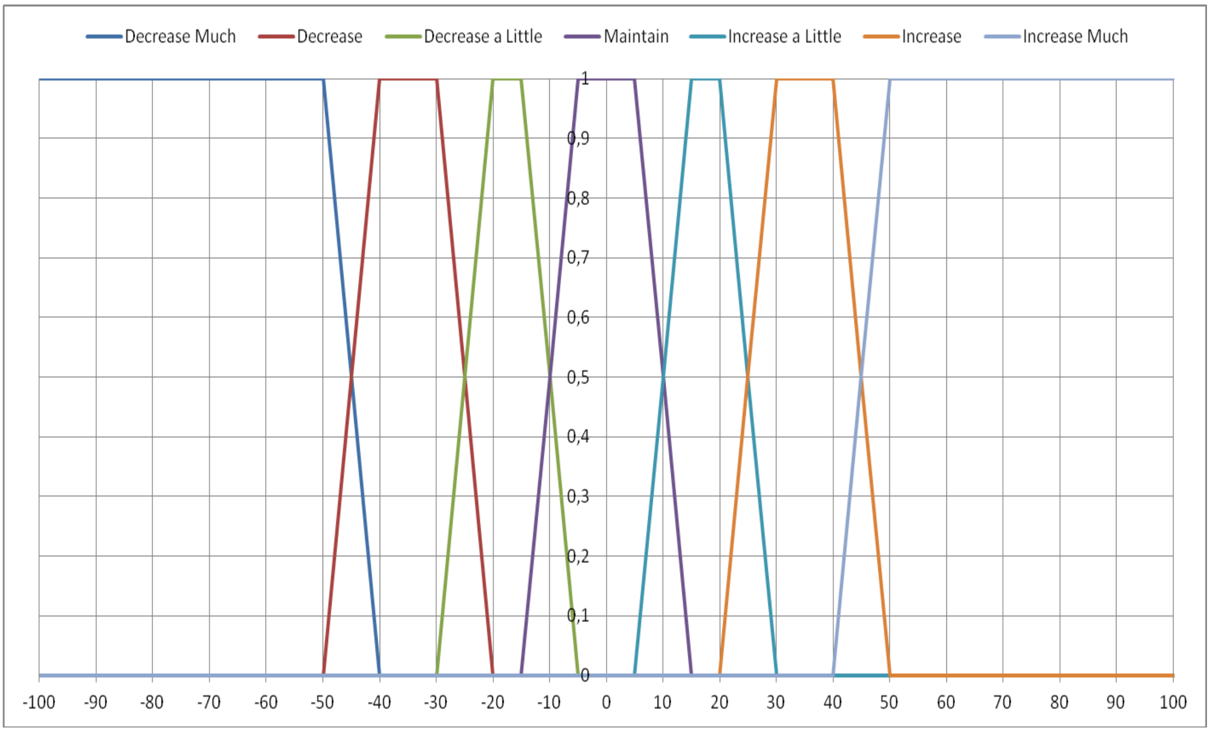

Figure 5. Membership Function – Variation (Budget, Ability)

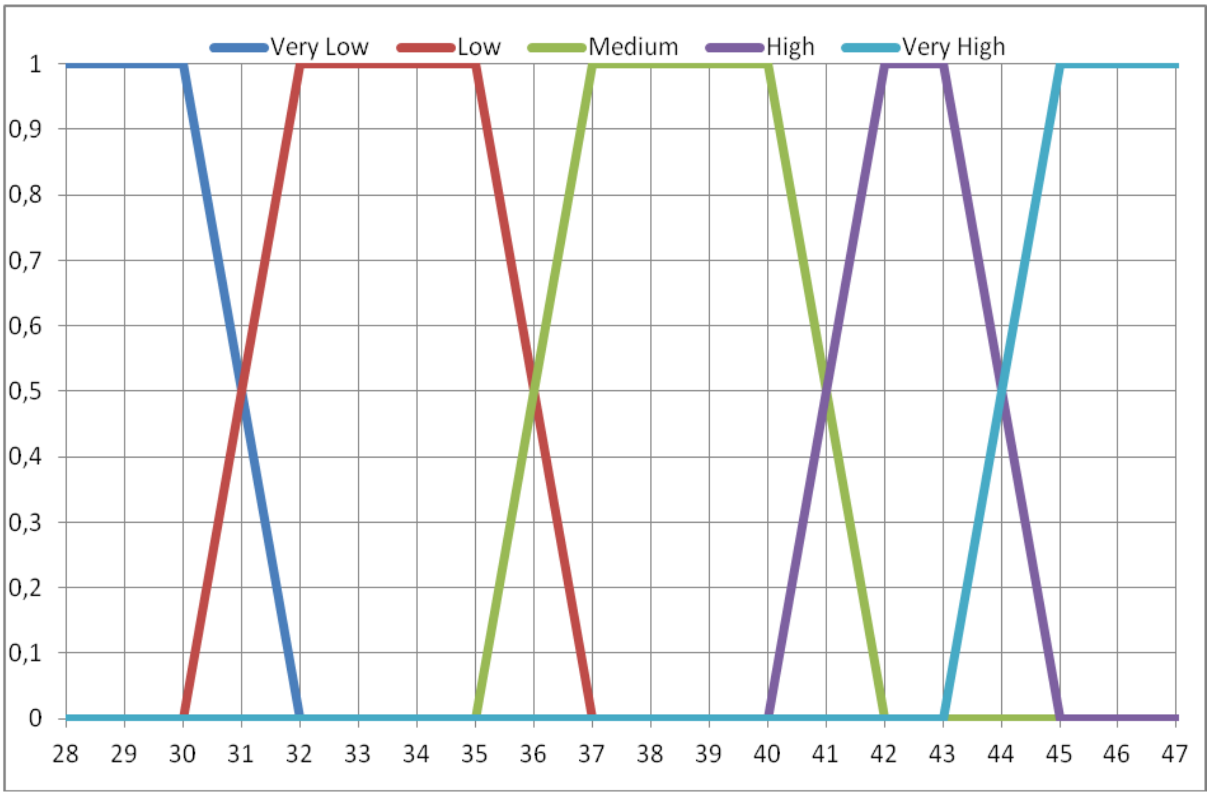

Figure 6. Membership Function – Variation (Budget, Ability)

The hours attribute cannot be represented using a percentage variation but only by a crisp value. Therefore a new set of fuzzy variables describing the amount of hours spent on training was proposed (Figure 6). In the most recent model there are 18 nodes with the attribute of Hours. Their membership functions defining fuzzy sets differ depending on nodes.

2.4. Relationships and Rules

Most relations in the model are defined using **fuzzy causal relationships (IF – THEN rules)**. Each rule models changes in the attribute of the "causal" node and its impact on the attribute of the "effect" node. This was the most common type of relationships used in the initial and new model.

IF Current Budget allocated to Operating Surfaces is Decreased Little
THEN Ability of Infrastructure DLoD will Decrease Much

As was mentioned previously, new relationships had to be introduced to reflect dependencies between some nodes. The model operates not only using causal relations but also linear ones with parameters defining the increase/decrease ratio.

Conditional fuzzy causal relationships exist between two nodes (where the second node is additionally considered when determining the effect value) and the "effect" node.

IF Current Budget allocated to Operating Surfaces is *Maintained*
AND NOT (Available Equipment is *Decreased* or *Decreased Much*)
THEN Ability of Infrastructure DLoD will *Decrease*

Both of the above mentioned relationships have been used in the first part of the project and a description of how they work (accumulation of impacts, etc.) can be found in the first report[3] (Chapters 3.1 and 3.4).

During the first workshop the experts identified more complex, conditional relationships, where more than one conditional node was involved. An example of such a relationship is in Figure 7. They operate the same way as simple conditional relationships.

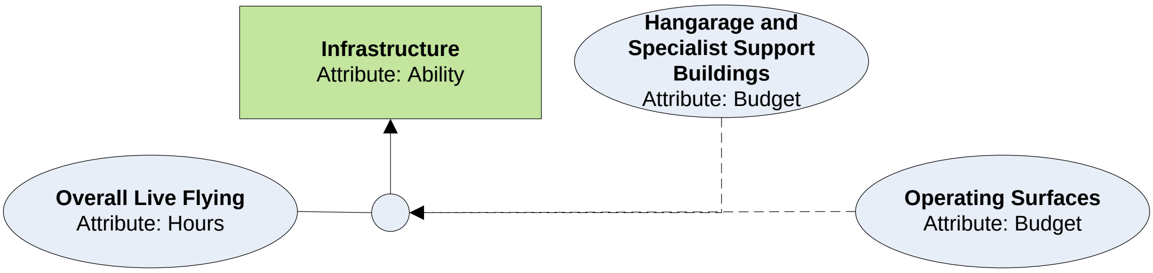

Figure 7. Two conditional nodes affecting Infrastructure DLoD

Example of rules defining **complex conditional relationships**:

IF Number of Hours allocated to Overall Live Flying is *Medium*
AND NOT (Operating Surfaces is Increased or Increased Much)
AND NOT (Hangarage and Specialist Support Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Number of Hours allocated to Overall Live Flying is *Medium*
AND NOT (Operating Surfaces is Increased or Increased Much)
AND (Hangarage and Specialist Support Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Number of Hours allocated to Overall Live Flying is *Medium*
AND (Operating Surfaces is Increased or Increased Much)
AND NOT (Hangarage and Specialist Support Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Decrease Little*

IF Number of Hours allocated to Overall Live Flying is *Medium*
AND (Operating Surfaces is Increased or Increased Much)
AND (Hangarage and Specialist Support Buildings is Increased or Increased Much)
THEN Ability of Infrastructure DLoD will *Maintain*

Applied modifications to the software allow having as many conditional nodes as is required. Also more than two conditional states of the conditional node can be considered in the rule. The following example does not exist in the model and is presented only to demonstrate the possibilities of building the rules.

IF Number of Hours allocated to Overall Live Flying is *Medium*
AND NOT (Operating Surfaces is **Increased Little or Increased or Increased Much**)
AND (Hangarage and... is Increased or Increased Much)

[3] Petrovic, D and Zdanowicz, P *Research and Development into Modelling and Analysis of DLoDs Using Causal Maps,* DSTLX1000068169, October 2012.

THEN Ability of Infrastructure DLoD will *Decrease Little*

Nodes representing synthetic training and live flying of a squadron (or OCU students/OCU staff) required not only a new type of attribute (Hours) and membership functions but also new type of relationships. As both types of trainings are equally important when the impact on Training DLoD's ability is considered therefore conditional relationships could not be used to represent this relation. To overcome this limitation a **complex fuzzy causal relationship** has been used. It assumes that both "causal" nodes are equally important and all combinations of values of both nodes have to be defined to represent the relationship. For two nodes with attribute Hours (five fuzzy variables) 25 rules had to be created.

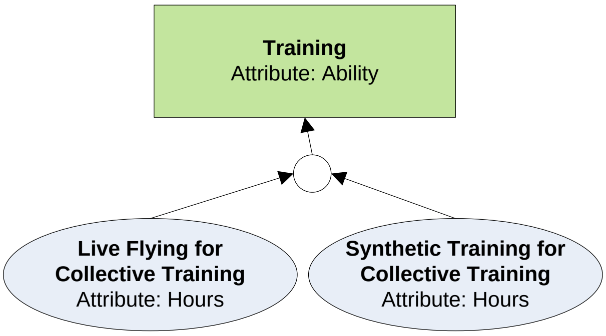

Figure 8. Complex fuzzy causal relationship

Examples of rules defining complex fuzzy causal relationship:

IF Number of Hours for Live Flying is *Very Low* and
Number of Hours for Synthetic Training is *Very Low*
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours for Live Flying is *Very Low* and
Number of Hours for Synthetic Training is *Low*
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours for Live Flying is *Very Low* and
Number of Hours for Synthetic Training is *Medium*
THEN Ability of Training DLoD will *Decrease*

IF Number of Hours for Live Flying is *Very Low* and
Number of Hours for Synthetic Training is *High*
THEN Ability of Training DLoD will *Decrease Little*

IF Number of Hours for Live Flying is *Very Low* and
Number of Hours for Synthetic Training is *Very High*
THEN Ability of Training DLoD will *Decrease Little*

IF Number of Hours for Live Flying is *Low* and
Number of Hours for Synthetic Training is *Very Low*
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours for Live Flying is *Low* and
Number of Hours for Synthetic Training is *Low*
THEN Ability of Training DLoD will *Decrease Much*

IF Number of Hours for Live Flying is *Low* and
Number of Hours for Synthetic Training is *Medium*
THEN Ability of Training DLoD will *Decrease*

In a final model **linear relationships** also exist. They are being used to sum the number of hours spent on Live Flying and Synthetic Training or to calculate the budget spent on Consumables and AAR, which is the number of Live Flying multiplied by their costs.

2.5. Accumulation of impacts

As it can be seen in Figure 3, each DLoD is affected by more than one causal node; this necessitates impacts to be accumulated.

For example, if there are 2 rules as follows:

IF Budget for Training Equipment is Increased THEN Training Ability is Increased Much
IF Budget for Training Methodology is Increased THEN Training Ability is Increased Little,
the accumulated impact of changes in budgets for Training Equipment and Training Methodolgy on the Training Ability should be *slightly higher than Increase Much.* However, the standard fuzzy mechanism would give a result equal to the linguistic term *Increase Much* (the higher value of the two). This reasoning mechanism is not appropriate for FCMs. Therefore, we implemented a Fuzzy Accumulation Reasoning algorithm as proposed in (Carvalho and Tome, 2009) to accumulate multiple impacts.

2.6. Software

For the purpose of the project a software performing analysis of different scenarios was developed. It allows an analysis to be performed on two different locations the squadron could be based in. The software needs two databases with different sets of rules. They are loaded when application starts, therefore different locations (than Marham and Lossiemuth) can be investigated if necessary.

3. Outcome

The GODIVA model was completed and, as demonstrated to and assessed by the second Dstl experts workshop, proved potentially[4] capable of successfully assessing the impact of the JSF base location options. A new development was the ability to assess the two options against differing criteria: capability and costs. The model also demonstrated potential to explore the effects of different relative levels of Live Flying and Synthetic Training.

4. Future work

During the second phase of the project it was demonstrated that the proposed methodology can manage the complexity of military reality. During one month of work on the extension of the causal map the model had significantly changed and this met with the approval of the board of military experts who took part in the second workshop.

In future work it will be possible to add new types of attributes, relations and more layers to further expand the map and allow easier translation of the reality to the mathematical model. It has to be remembered, though, that the model should be kept as simple as possible to adequately address the problem at hand, without unnecessary details.

Another important improvement that could be considered is an alternative representation of DLoDs and Force Element’s ability. At the moment it is represented as a hypothetical value between 0 and 100. Experts who took part in workshops agreed that such a representation is too abstract and can be misleading. It will be better if it is replaced by more meaningful concepts. The following changes could be done: instead of Equipment DLoD, a number of airframes ready to fight and fully equipped; instead of Training DLoD, the number of pilots that are skilled enough to perform military tasks, etc.

Apart from further improvements of the model, an optimisation of the resources allocation could be done. One of the methods that could be used is a genetic algorithm or different AI optimisation

[4] We can only say “potentially” since, for classification reasons, this project did not use genuine data regarding the two bases but an illustrative data set.

techniques. It will look for the most efficient resource allocation in respect to military ability and overall budget. These parameters will be adjustable, allowing analysis of different scenarios and will help to make final decisions. So far a user has to modify the budget and try to find the best solution by running many simulations.

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