

# Concentration and Asymmetry in Air Power

Historical lessons for the defensive employment of small air forces



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What do the **historical data** tell us about the nature of concentration in air combat?

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What can a **historical analysis** tell us about the implications for tactical and operational principles?

## Lanchester's Square Law (1914)

The **aimed-fire** model:  $G(t)$  Green units fight  $R(t)$  Red units.

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and vice versa. Divide:

$$\frac{dR}{dG} = \frac{gG}{rR} \quad \text{or} \quad rR dR = gG dG$$

and integrate:

$$\frac{1}{2}rR^2 = \frac{1}{2}gG^2 + \text{constant}$$

throughout the battle, the **Square Law**.

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suppose we begin with twice as many Reds as Greens,  $R_0 = 2G_0$ ,  
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Then

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## Concentration is good:

If Red divides its forces, and Green fights each half in turn,  
Green wins the first battle, with  $\sqrt{2/3} \simeq 80\%$  of  $G_0$  remaining,  
Green wins the second battle, with  $\sqrt{1/3} \simeq 60\%$  of  $G_0$  remaining.



## Lanchester's Linear Law

**Ancient warfare**, along a fixed, narrow battle-line with  $N(t)$  fighting on each side:

$$\frac{dG}{dt} = -rN \quad \frac{dR}{dt} = -gN$$

**Modern warfare**, but with hidden targets (the **unaimed-fire** model):

$$\frac{dG}{dt} = -rRG \quad \frac{dR}{dt} = -gGR$$

Either way,  $dR/dG$  is now fixed, and the constant quantity is

$$rR - gG,$$

the more intuitive **linear law**: fighting strength is just numbers  $\times$  effectiveness.

# Asymmetric warfare

Green attacks, Red defends:

$$\frac{dG}{dt} = -rR, \quad \frac{dR}{dt} = -gG\frac{R}{K}$$

where  $K$  is a constant parameter. Then

$$rRK - \frac{1}{2}gG^2$$

is conserved, so that

- Green benefits more from numbers and concentration, but
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**Red has a defender's advantage**

## Generalized scaling laws for air combat

Fit loss-rates to powers of own and enemy numbers:

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Divide, re-arrange, integrate: we find that

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is constant, where  $\rho = 1 + r_1 - r_2$  and  $\gamma = 1 + g_1 - g_2$ ,  
the **exponents**, capture the conditions of battle:

- Green should concentrate its force if  $\gamma > 1$ , divide if  $\gamma < 1$ .
- if  $\rho > \gamma$  then Green has a defender's advantage, by a factor  $\rho/\gamma$

# Symmetric dynamics: The loss and force ratios

The crucial tactical relationship is

$$\frac{dG}{dR} = \frac{r R^{\rho-1}}{g G^{\gamma-1}}.$$

If the dynamics are **symmetric**,  $\rho = \gamma$ , we can ask:

How does the **loss ratio**  $dG/dR$  depend on the **force ratio**  $R/G$  ?

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Two obvious possibilities are

**Lanchester's square law**: simple proportionality,  $\rho = \gamma = 2$

**Lanchester's linear law**: no dependence



## Symmetric dynamics: The loss and force ratios

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Cites a 1970 study of Korea and WW2.

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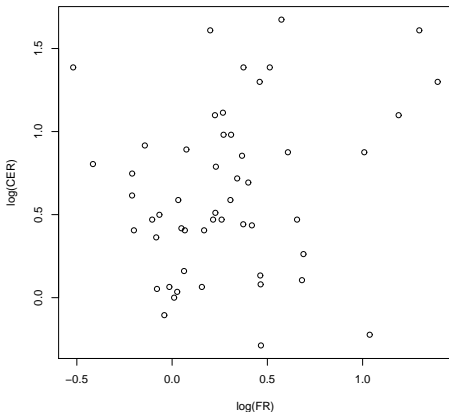
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Well, no.

NJM, *Is air combat Lanchestrian?*, *MORS Phalanx* **44**, no. 4 (2011) 12-14

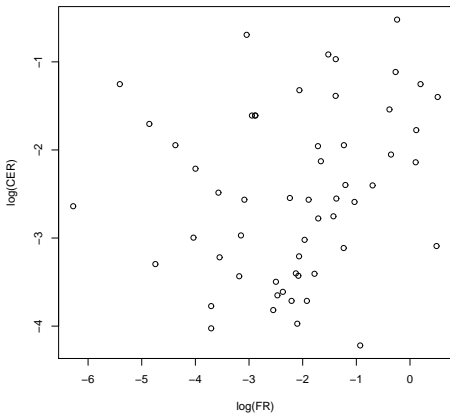
# The loss ratio: Battle of Britain



$\log dG/dR$  vs  $\log R/G$

$G$ =Luftwaffe,  $R$ =Royal Air Force

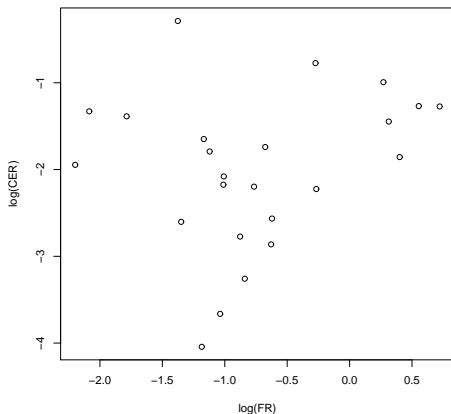
# The loss ratio: Pacific air war



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$G$ =Americans,  $R$ =Japanese

## The loss ratio: Korea



$\log dG/dR$  vs  $\log R/G$

$G$ =Americans,  $R$ =KPAF/Chinese

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To the extent to which it obeys a symmetric Lanchester law,  
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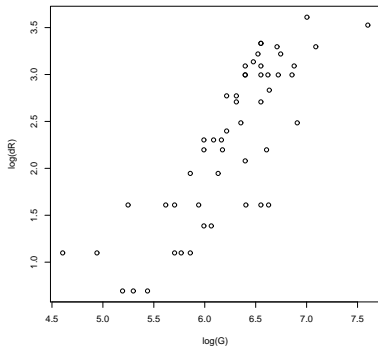
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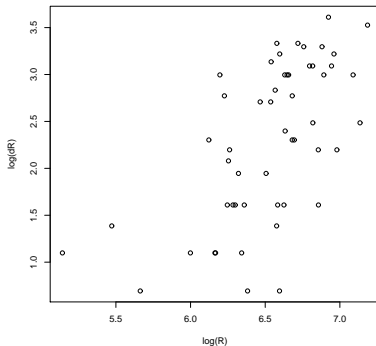
But air combat is **asymmetric**.



# Battle of Britain: RAF losses



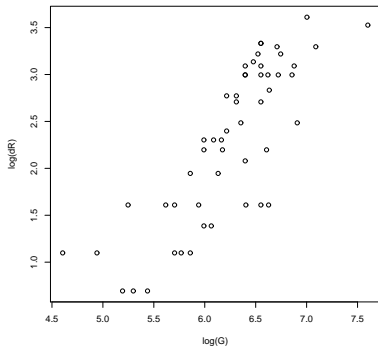
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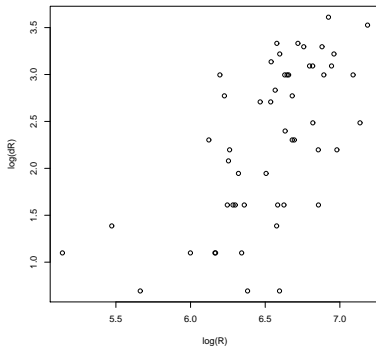
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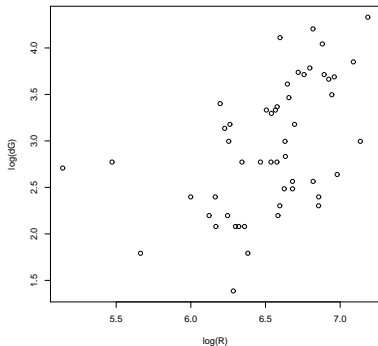


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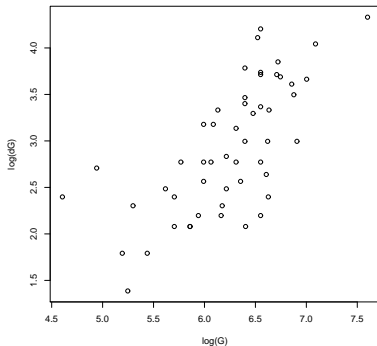
$$\frac{dR}{dt} = -gG^{1.12 \pm 0.17} R^{0.18 \pm 0.25}$$

Hooray for Lanchester!

# Battle of Britain: Luftwaffe losses



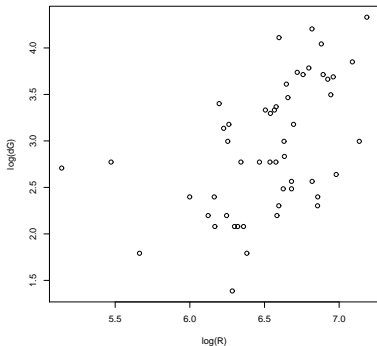
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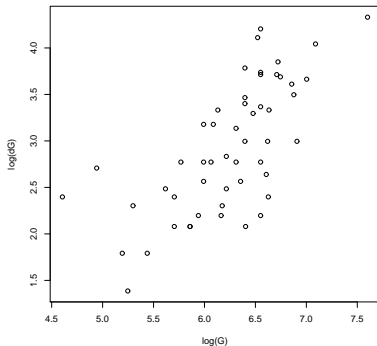
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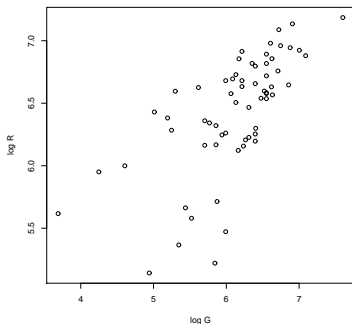
$\log \delta G$  vs  $\log G$

$$\frac{dG}{dt} = -rR^{0.00 \pm 0.25} G^{0.86 \pm 0.18}$$

Not so good.

# Subtleties

$G$  and  $R$  are highly correlated (0.74):



$\log R$  vs  $\log G$

and so the overall powers in the loss-rates,  $g_1 + r_2$  and  $r_1 + g_2$ , are better-determined than their constituents: variation is less significant *along* the lines of constant  $g_1 + r_2$  and  $r_1 + g_2$  than *orthogonal* to them.

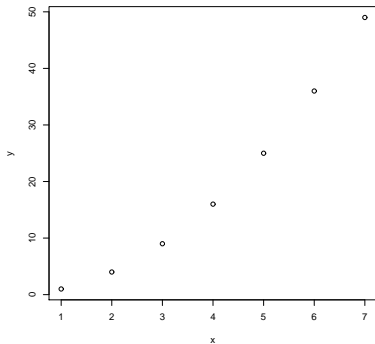
## Subtleties

When  $g_1 + r_2 \neq 1$  or  $r_1 + g_2 \neq 1$ , autonomous battles ('raids') should not be aggregated into daily data.

If they are, the effect is to push the overall powers  $g_1 + r_2$  and  $r_1 + g_2$  away from their true values and towards one, and to reduce the quality of the fit.

# Subtleties

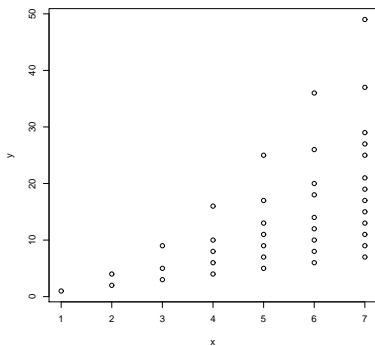
**Example:**  $y = x^2$



has  $\log y = 2 \log x$ , of course.

## Subtleties

**Example:**  $y = x^2$  and sums of these: e.g. not only  $(3, 9)$  but also  $(1 + 2, 1 + 4) = (3, 5)$  and  $(1 + 1 + 1, 1 + 1 + 1) = (3, 3)$ .



and the best fit is now  $\log y = 1.5 \log x$ , with  $\Sigma R^2 = 0.6$ .



# Subtleties

Upshot: **asymmetry** is typically **greater** than the data suggest.

## The Battle of Britain: Overall

$$\frac{dR}{dt} = -gG^{1.12 \pm 0.17} R^{0.18 \pm 0.25}, \quad \frac{dG}{dt} = -rR^{0.00 \pm 0.25} G^{0.86 \pm 0.18}$$

has  $\gamma \equiv 1 + g_1 - g_2 \simeq 1.3$ ,  $\rho \equiv 1 + r_1 - r_2 \simeq 0.8$ .

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More accurate are the differences of  $g_1 + r_2$  or  $r_1 + g_2$  from one:

$$g_1 + r_2 = 1.30, \quad r_1 + g_2 = 0.86,$$

and thus the **asymmetry**

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We can conclude with fair confidence that  $\gamma > 1$  and  $\rho < 1$ , and with much more confidence that  $\gamma > \rho$ .

Thus the German attackers may have benefited from mere numbers, all else equal: but the British defenders did not.

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**No**



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Rather, to the extent to which  $\gamma > \rho$ , the RAF had a defender's advantage.

The achievement of Keith Park (Commander, 11 Group, RAF Fighter Command) lay in creating and exploiting this advantage:

*'It [is] better to have even one strong squadron of our fighters over the enemy than a wing of three climbing up below them'*

NJM & Chris Price, *Safety in Numbers: Ideas of concentration in Royal Air Force fighter defence from Lanchester to the Battle of Britain*, *History* **96** (2011) 304-325.

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# Vietnam 1965-68; Rolling Thunder

Engagement-level data, and a simple linear regression of loss rates against numbers.

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NVAF (MiG 17,19,21) sorties tend to cause **own** losses, whether against F4s or F105s.

**NVAF conclusion:** sortie sparingly, disrupt, avoid engagement.

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air combat is approximately linear-law, and **asymmetric**.

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air combat is approximately linear-law, and **asymmetric**.

To the extent to which there is some advantage in numbers, this is true only for the **attacker**. In contrast the **defender's** optimal tactics are of cover, concealment, dispersal, denial, disruption, force preservation.

Ian Horwood, NJM & Chris Price, Concentration and asymmetry in defensive air combat: from the battle of Britain to the 21st century, submitted to *Air Power Review*.